Regents Park Publishers

Operations Management

T1LM4

Tutorials

Content

Content

Axioms

Axioms

An **Axiom** is a mathematical statement that is **assumed** to be true.

- a self-evident truth that requires no proof
- a universally accepted principle or rule
- a proposition that is assumed without proof for the sake of studying the consequences that follow from it
- Formulas that we use are based on axioms

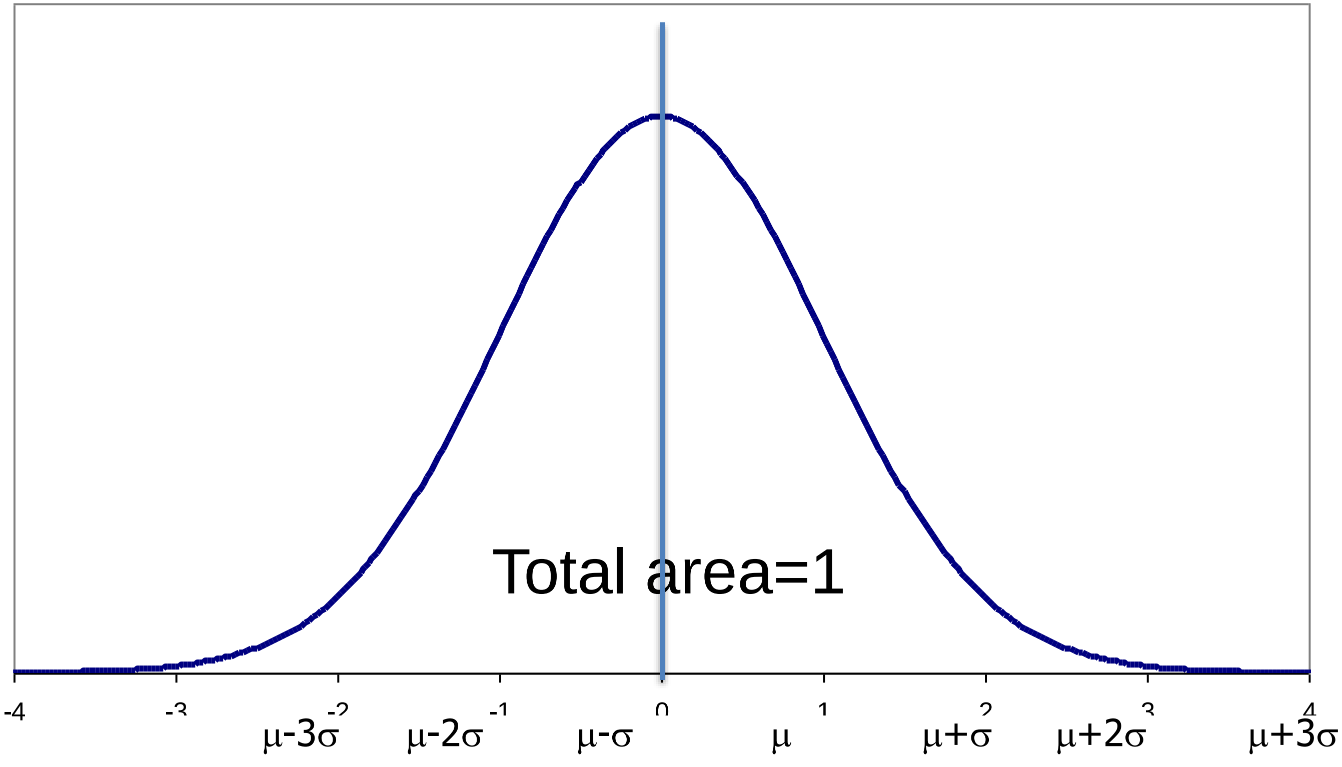
Normal Probability Distribution

Introduction

- The Normal (or Gaussian) distribution is probably the most used (and abused) distribution in statistics.

- Normal random variables are **continuous** (they can take any value on the real line) so the Normal distribution is an example of a **continuous probability distribution.**

The Normal Distribution



The graph of the normal distribution is called the normal (or bell) curve.

The Normal Distribution

- The mean, median, and mode are the same.
- The normal curve is symmetric about its mean.
- The total area under the normal curve is one.
- The normal curve approaches, but never touches, the x-axis.

Normal Distributions and Probability

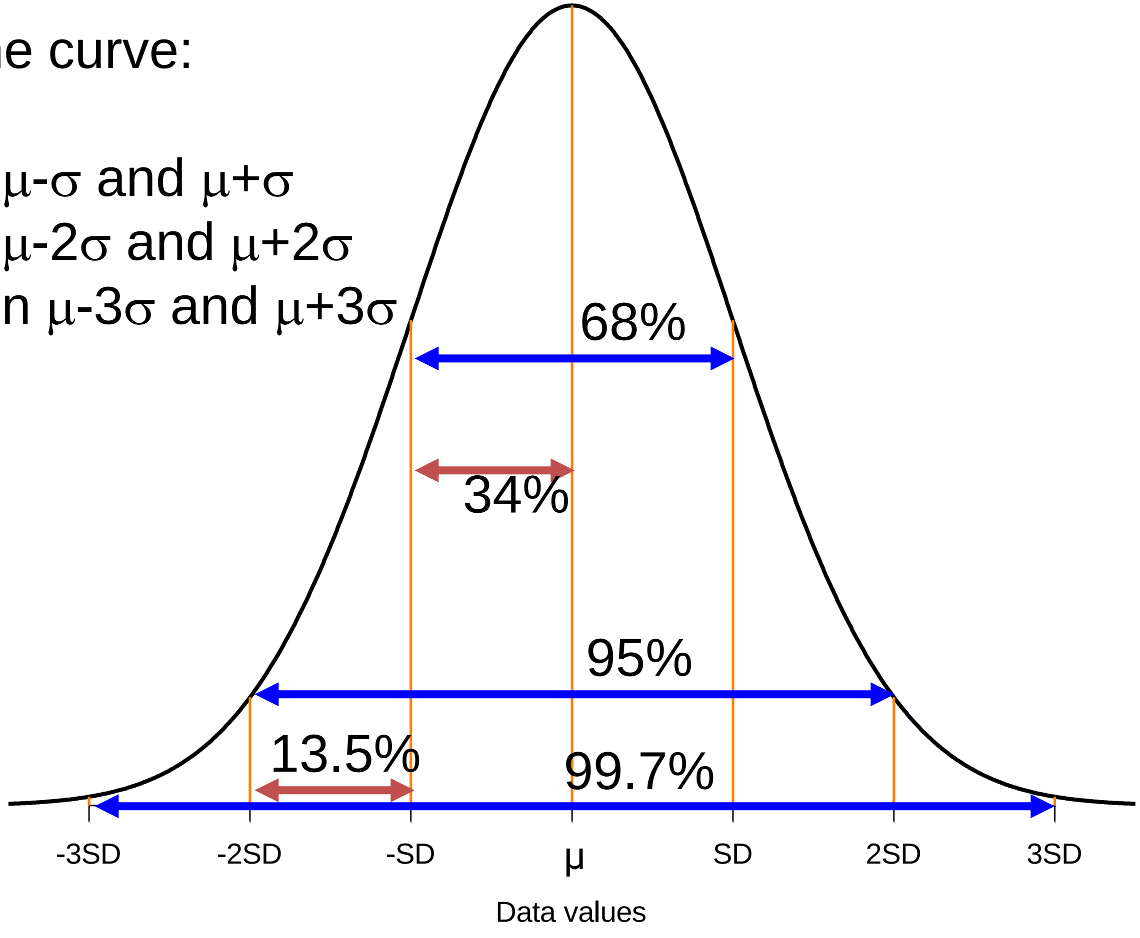
- We do not talk about the probability $P[X=x]$ for continuous random variables. Rather we talk about the probability that the random variable falls in an interval, i.e. $P[x_1 \leq X \leq x_2]$.

- $P[x_1 \leq X \leq x_2]$ can be determined by finding the area under the normal curve between x_1 and x_2.

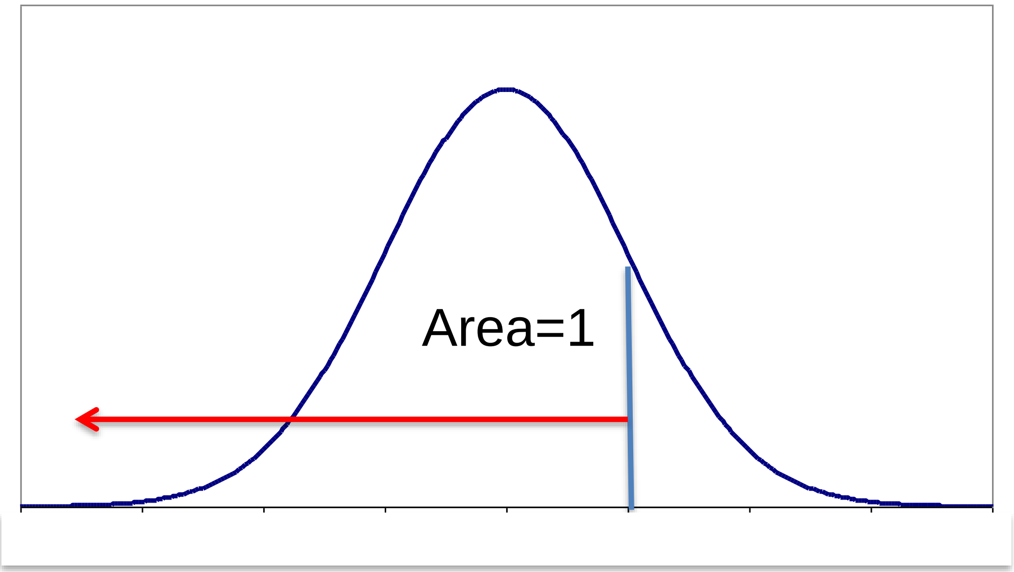
Normal Distributions and Probability

The areas under the curve:

68% lies between $\mu-\sigma$ and $\mu+\sigma$
95% lies between $\mu-2\sigma$ and $\mu+2\sigma$
99.7% lies between $\mu-3\sigma$ and $\mu+3\sigma$

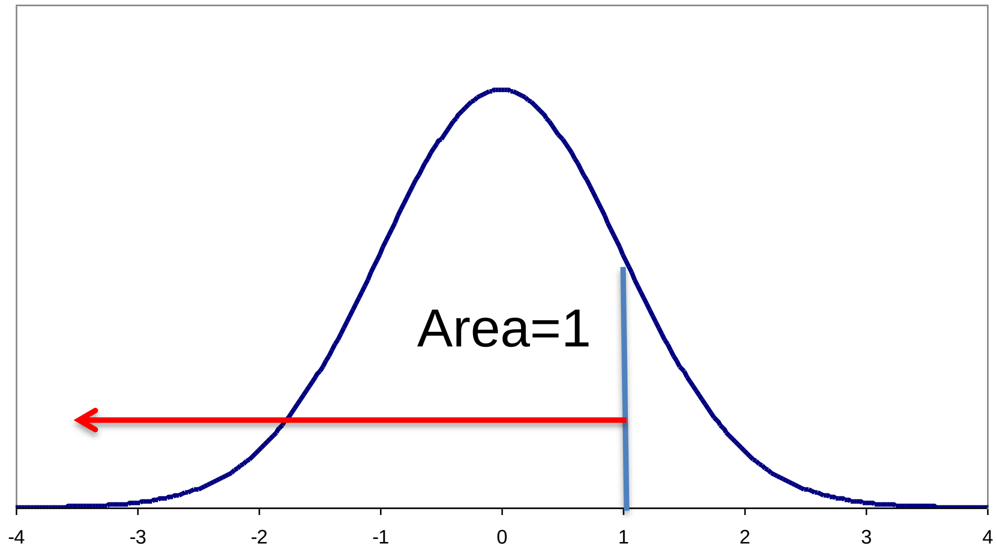


Excel Function



The area under the normal distribution
from x to -∞ can be computed using the EXCEL function
NORMDIST

Excel Function



The area under the standard normal distribution from x to -∞ can be computed using the EXCEL function **NORMSDIST**

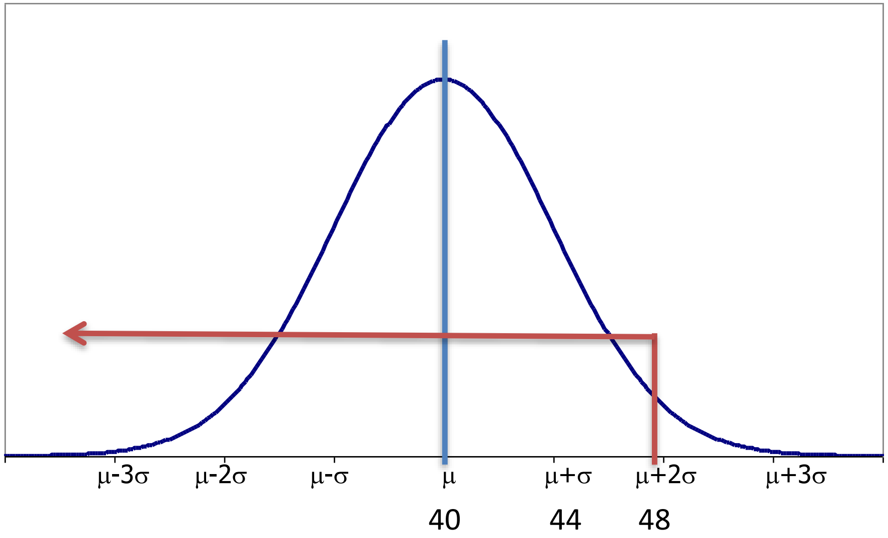
Examples 1-3

Example 1

- The mean length of a fish is 40cm and the standard deviation is 4 cm. What is the probability that the length of a randomly selected fish is less than 48cm?
- 48cm is two standard deviations above the mean so the area to the left of 48cm is 0.5+0.475 = **0.9772**

or

NORMDIST (48,40,4,1)= 0.9772



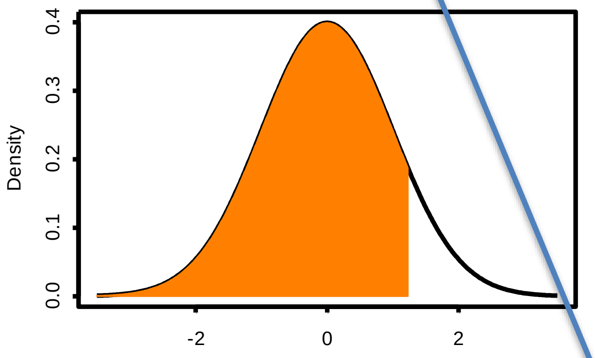
Example-2

- Find the area under the normal distribution curve between -0.12 and 1.23.
  - We use our previous approach:
    - $P[-0.12 \le X \le 1.23] = P[X \le 1.23] - P[X \le -0.12]$
  - In EXCEL:
    - NORMDIST(1.23,0,1,1)-NORMDIST(-0.12,0,1,1)

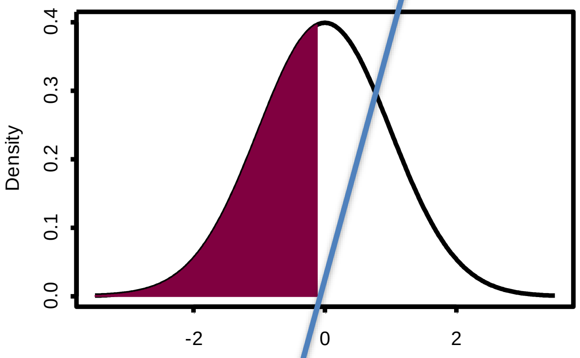
NORMDIST(x,μ,σ,1)

Example-3

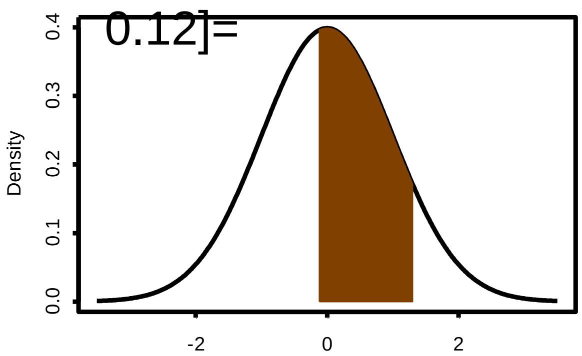
P[X≤1.23]= **0.8907**



P[X≤-0.12]= **0.4522**



P[X≤1.23]-P[X≤-0.12]=



0.8907-0.4522 = 0.4384

Standardized Normal Probability Distribution

The Standard Normal Distribution

The normal distribution with a mean of 0 and a standard deviation of 1 is called the

Standard Normal Distribution

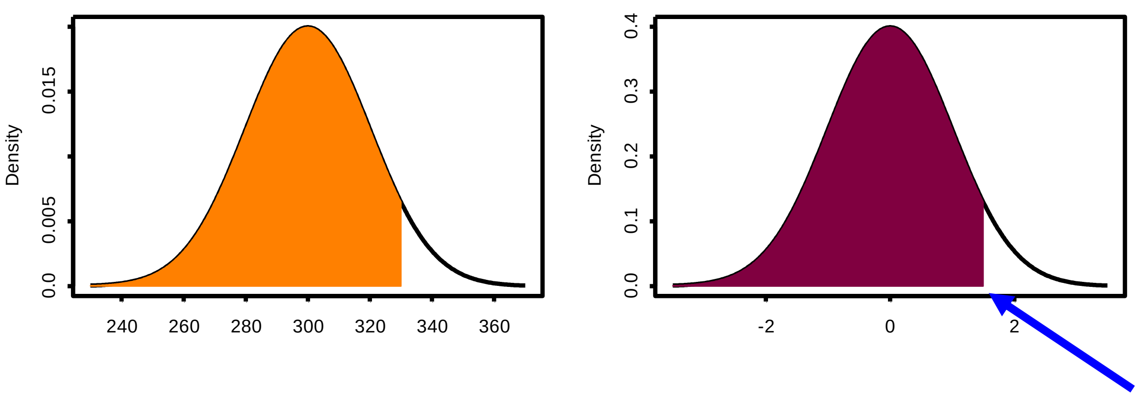
- The standard normal distribution and the z-score:

$$z = \frac{\text{value - mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

Standardization

- We can transform any normal distribution into a standard normal distribution by subtracting the mean and dividing by the standard deviation.

μ=300; σ=20;x=330 μ=0; σ=1;z=1.5



Area=0.933 in both cases

Z = (x-300)/20

Standardization

- To find the probability that $X \leq Y$ if X is normally distributed with mean μ and standard deviation σ.
  - Compute the z-score: $z = (y - \mu)/\sigma$
  - Calculate the area under the normal curve between $-\infty$ and z
  - We could calculate this area directly using the EXCEL function:

Standardize

Examples 1-3

Examples 1-3

- The average swimming speed of a fish population is 2 m.s. (standard deviation 0.5). You select a fish at random. What is the probability that:
  - Its swimming speed is less than 1 m.s.
  - Its swimming speed is greater than 2.5 m.s.
  - Its swimming speed is between 2 and 3 m.s.

Example -1

- The average swimming speed of a fish population is 2 m.s. (standard deviation 0.5). You select a fish at random. What is the probability that:

  – Its swimming speed is less than 1 m.s.

    $= P(z < (1-2)/.5) = P(z < -2) =$ **0.0228**

Examples-2

- The average swimming speed of a fish population is 2 m.s. (standard deviation 0.5). You select a fish at random. What is the probability that:
  - Is swimming speed is greater than 2.5 m.s.

$= P(z > (2.5-2)/.5) = P(z > 1) = 1 - P(z \leq 1)$

= 0.159

Example-3

- The average swimming speed of a fish population is 2 m.s. (standard deviation 0.5). You select a fish at random. What is the probability that:
  - Its swimming speed is between 2 and 3 m.s.
  - = P((2-2)/.5 ≤ z ≤ (3-2)/0.5)
    = P(0 ≤ z ≤ 2) = P(z ≤ 2) – P(z ≤ 0)

= 0.477

Time Value of Money

The Role of Time Value in Finance

- Most financial decisions involve costs & benefits that are spread out over time.
- Time value of money allows comparison of cash flows from different periods.

Question

Would it be better for a company to invest $100,000 in a product that would return a total of $200,000 after one year, or one that would return $220,000 after two years?

The Role of Time Value in Finance (cont.)

- Most financial decisions involve costs & benefits that are spread out over time.
- Time value of money allows comparison of cash flows from different periods.

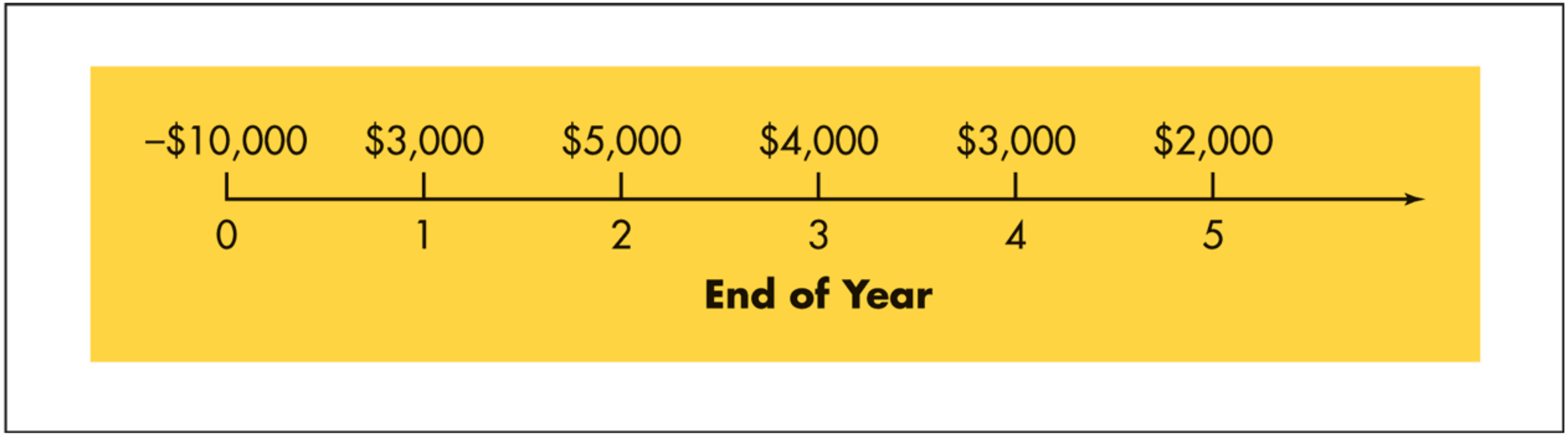
Answer

It depends on the interest rate!

Basic Concepts

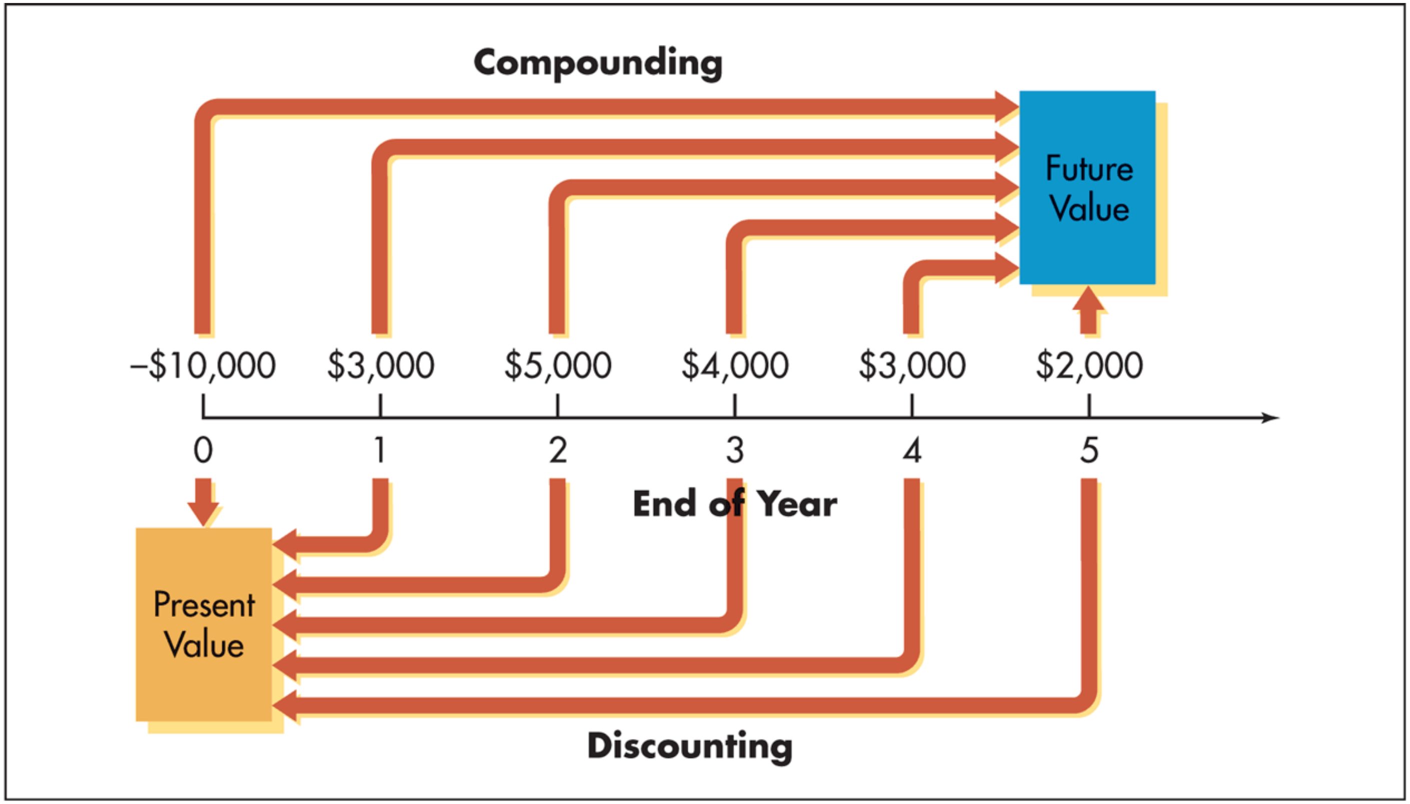
- **Future Value**: compounding or growth over time
- **Present Value**: discounting to today's value
- **Single** cash flows & **series** of cash flows can be considered
- **Timelines** are used to illustrate these relationships

Computational Aids



Time Line
Time line depicting an investment's cash flows

Computational Aids (cont.)



Compounding and Discounting
Time line showing compounding to find future value and discounting to find present value

Basic Patterns of Cash Flow

- The cash inflows and outflows of a firm can be described by its general pattern.
- The three basic patterns include a single amount, an annuity, or a mixed stream:

	Mixed cash flow stream	
End of year	A	B
1	$ 100	−$ 50
2	800	100
3	1,200	80
4	1,200	− 60
5	1,400	
6	300	

Simple Interest

With simple interest, you don’t earn interest on interest.

- Year 1: 5% of $100 = $5 + $100 = $105
- Year 2: 5% of $100 = $5 + $105 = $110
- Year 3: 5% of $100 = $5 + $110 = $115
- Year 4: 5% of $100 = $5 + $115 = $120
- Year 5: 5% of $100 = $5 + $120 = $125

Compound Interest

- With compound interest, a depositor earns interest on interest:
- Year 1: 5% of $100.00 = $5.00 + $100.00 = $105.00
- Year 2: 5% of $105.00 = $5.25 + $105.00 = $110.25
- Year 3: 5% of $110.25 = $5 .51+ $110.25 = $115.76
- Year 4: 5% of $115.76 = $5.79 + $115.76 = $121.55
- Year 5: 5% of $121.55 = $6.08 + $121.55 = $127.63

Time Value Terms

PV_0	=	present value or beginning amount
i	=	interest rate
FV_n	=	future value at end of “n” periods
n	=	number of compounding periods

Future Value of a Single Amount

- **Future Value** techniques typically measure cash flows at the end of a project's life.
- Future value is cash you will receive at a given future date.
- The future value technique uses compounding to find the future value of each cash flow at the end of an investment's life and then sums these values to find the investment's future value.
- We speak of compound interest to indicate that the amount of interest earned on a given deposit has become part of the principal at the end of the period.

Example 1

Jane Farber places $800 in a savings account paying 6% interest compounded annually. She wants to know how much money will be in the account at the end of five years.

$$\mathbf{FV_5 = \$800 \times (1 + 0.06)^5 = \$800 \times (1.338)}$$

$$\mathbf{= \$1{,}070.40}$$

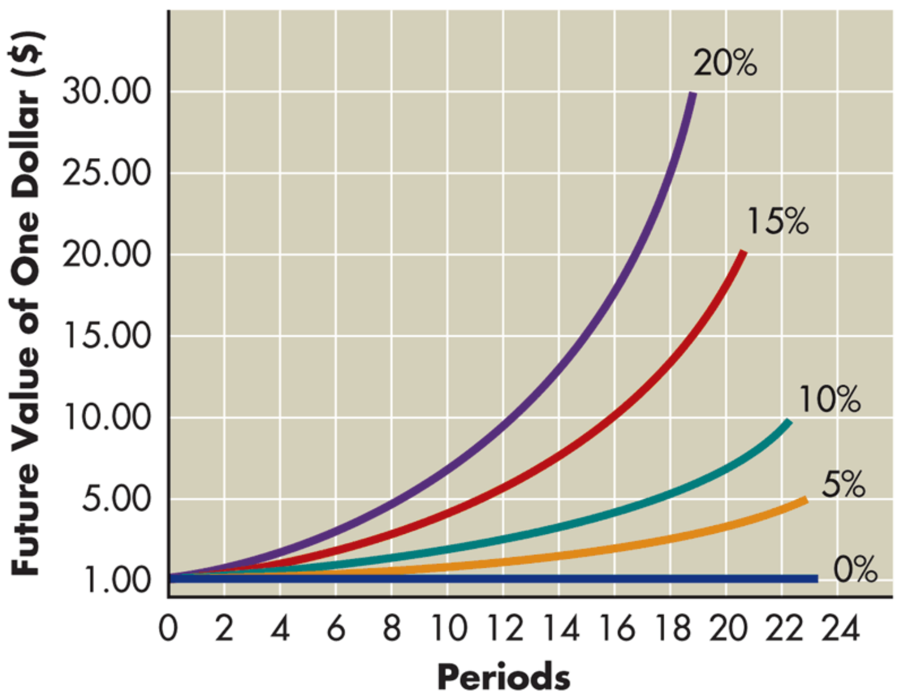
Future Value of a Single Amount Using Excel

	A	B
1	**FUTURE VALUE OF A SINGLE AMOUNT**	
2	Present value	$800
3	Interest rate, pct per year compounded annually	6%
4	Number of years	5
5	Future value	$1,070.58

Entry in Cell B5 is =FV(B3,B4,0,–B2,0).

The minus sign appears before B2 because the present value is an outflow (i.e., a deposit made by Jane Farber).

Future Value of a Single Amount: A Graphical View of Future Value



Future Value Relationship
Interest rates, time periods, and future value of one dollar

Present Value of a Single Amount

- **Present value** is the current dollar value of a future amount of money.
- It is based on the idea that a dollar today is worth more than a dollar tomorrow.
- It is the amount today that must be invested at a given rate to reach a future amount.
- Calculating present value is also known as **discounting**.
- The discount rate is often also referred to as the opportunity cost, the **discount rate**, the **required return**, or the **cost of capital**.

Example 2

Paul Shorter has an opportunity to receive $300 one year from now. If he can earn 6% on his investments, what is the most he should pay now for this opportunity?

$\$300 \times [1/(1.06)^1] = \$300 \times PVIF_{6\%,1}$

$\$300 \times 0.9434 = \$283.02$

Example 3

Pam Valenti wishes to find the present value of $1,700 that will be received 8 years from now.

Pam's opportunity cost is 8%.

$$\textbf{PV} = \$1{,}700/(1 + 0.08)^{8} = \$1{,}700/1.851$$

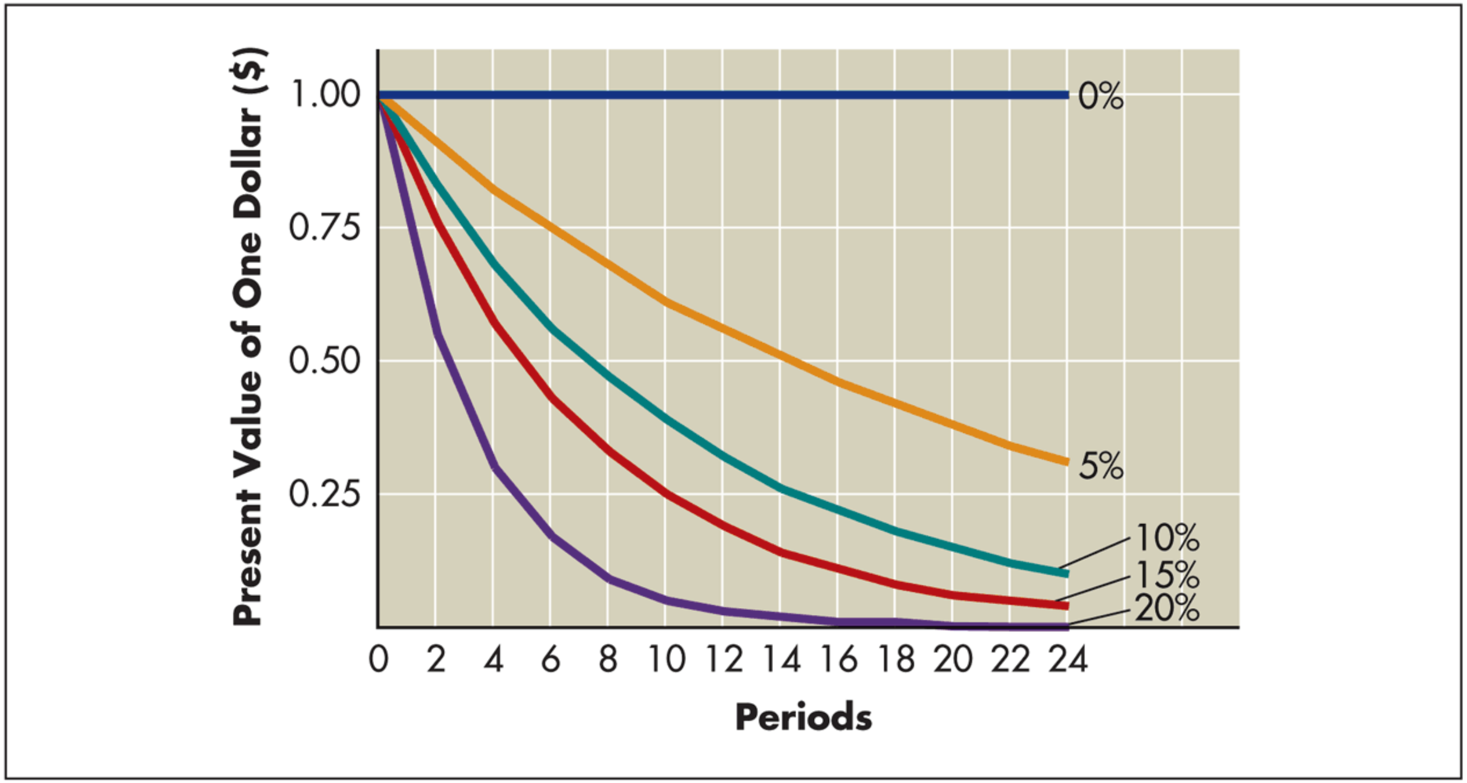
$$= \$918.42$$

Present Value of a Single Amount: Using Excel

	A	B
1	**PRESENT VALUE OF A SINGLE AMOUNT**	
2	Future value	$1,700
3	Interest rate, pct per year compounded annually	8%
4	Number of years	8
5	Present value	$918.46

Entry in Cell B5 is =–PV(B3,B4,0,B2).
The minus sign appears before PV to change the present value to a positive amount.

Present Value of a Single Amount: A Graphical View of Present Value



Present Value Relationship

Discount rates, time periods, and present value of one dollar

Future Value of a Mixed Stream: Using Excel

	A	B
1	**FUTURE VALUE OF A MIXED STREAM**	
2	Interest rate, pct/year	8%
3	Year	Year-End Cash Flow
4	1	$11,500
5	2	$14,000
6	3	$12,900
7	4	$16,000
8	5	$18,000
9	Future value	$83,608.15

Entry in Cell B9 is
=–FV(B2,A8,0,NPV(B2,B4:B8)).
The minus sign appears before FV to convert the future value to a positive amount.

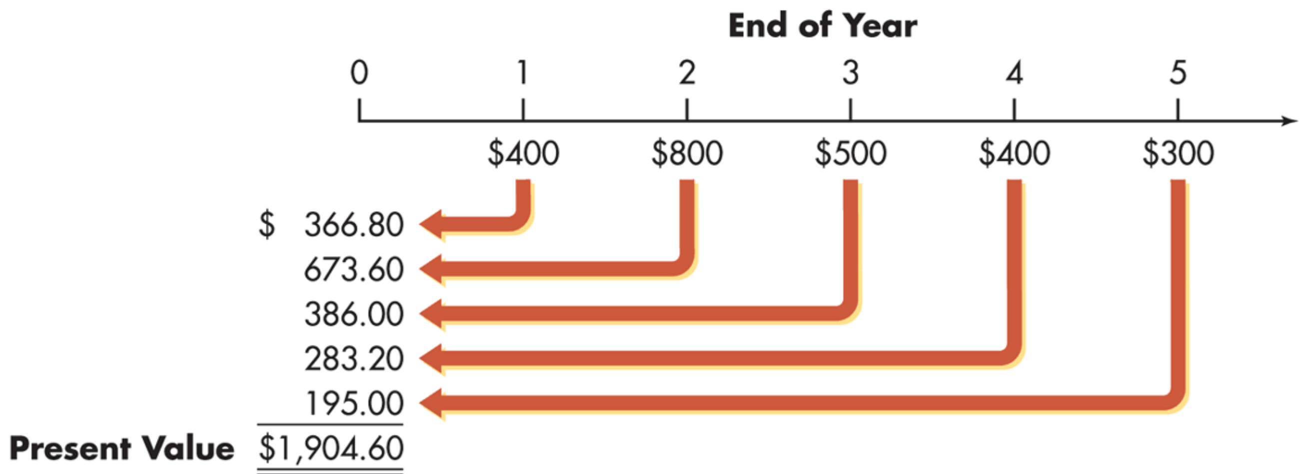
Example 4

Frey Company, a shoe manufacturer, has been offered an opportunity to receive the following mixed stream of cash flows over the next **5** years.

End of year	Cash flow
1	$400
2	800
3	500
4	400
5	300

Present Value of a Mixed Stream

- If the firm must earn at least **9%** on its investments, what is the most it should pay for this opportunity?
- This situation is depicted on the following time line.



Present Value of a Mixed Stream: Using Excel

	A	B
1	**PRESENT VALUE OF A MIXED STREAM OF CASH FLOWS**	
2	Interest Rate, pct/year	9%
3	Year	Year-End Cash Flow
4	1	$400
5	2	$800
6	3	$500
7	4	$400
8	5	$300
9	Present value	$1,904.76
	Entry in Cell B9 is =NPV(B2,B4:B8).	

Net Present Value

NPV as a Capital Budgeting Technique

Determining the discount value of series of future cash receipts is known as the Net Present Value.

Because capacity and process alternatives exist, so do options regarding capital investments and variable costs.

Managers must choose from among different financial options as well as capacity and process alternatives.

Analysts should show the capital investment, variable cost, and cash flow as well as net present value for each alternative.

Example of NPV as a Capital Budgeting Technique

Bennett Company is a medium sized metal fabricator that is currently contemplating two projects:

Project A requires an initial investment of $42,000.

Project B an initial investment of $45,000.

The relevant operating cash flows for the two projects are shown next.

Capital Budgeting Techniques (cont.) Bonus Quiz (in class)

Capital Expenditure Data for Bennett Company

	Project A	Project B
Initial investment	$42,000	$45,000
Year	Operating cash inflows	
1	$14,000	$28,000
2	14,000	12,000
3	14,000	10,000
4	14,000	10,000
5	14,000	10,000

Capital Budgeting Techniques (cont.)

Bennett Company's Projects A and B
Time lines depicting the conventional cash flows of projects A and B

Net Present Value

Net Present Value is found by subtracting the present value of the after-tax outflows from the present value of the after-tax inflows.

$$\text{NPV} = \sum_{t=1}^{n} \frac{CF_t}{(1+k)^t} - CF_0$$

$$= \sum_{t=1}^{n} (CF_t \times PVIF_{k,t}) - CF_0$$

Net Present Value (cont.)

Net Present Value is found by subtracting the present value of the after-tax outflows from the present value of the after-tax inflows.

Decision Criteria

If NPV > 0, accept the project

If NPV < 0, reject the project

If NPV = 0, technically indifferent

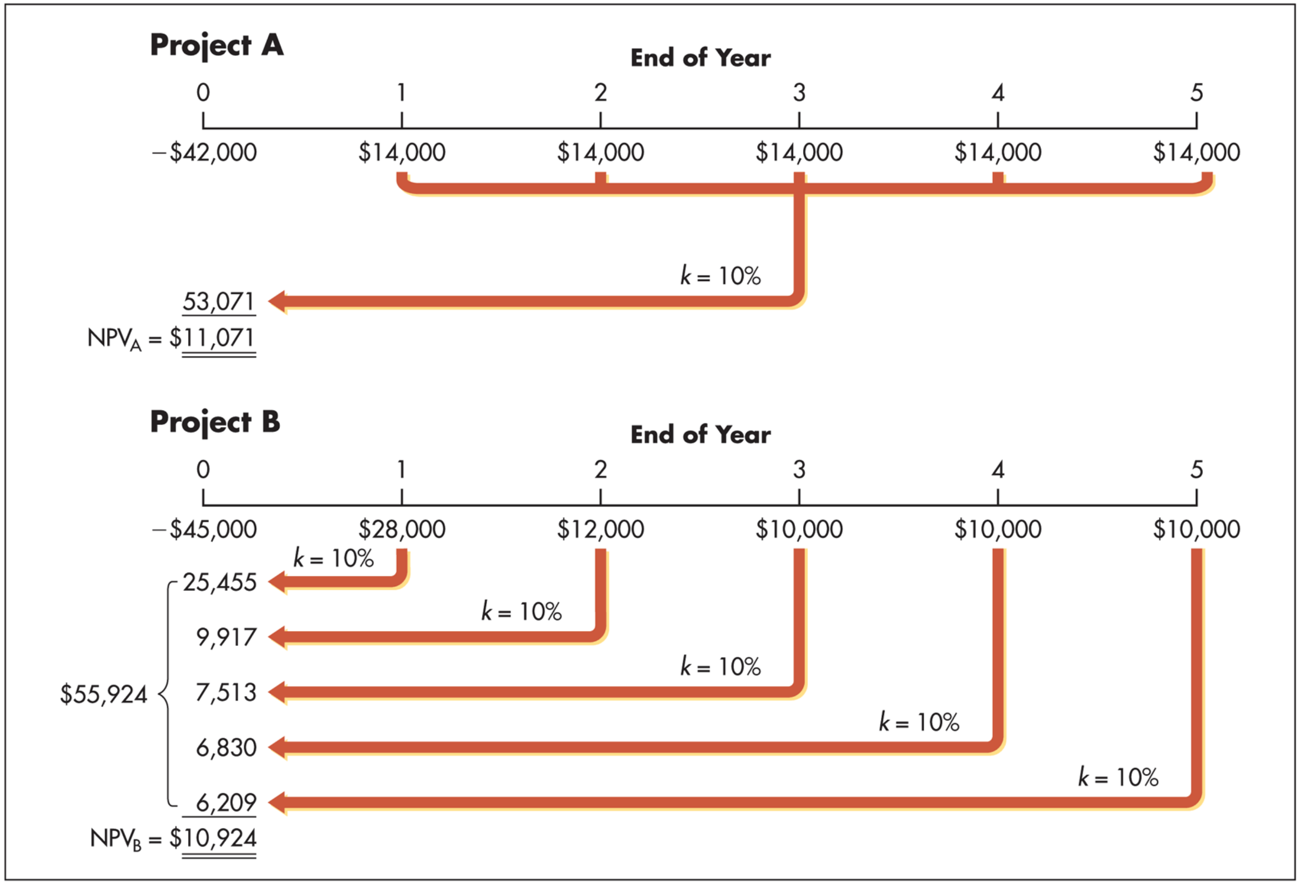
Net Present Value (cont.)

Using the Bennett Company data, assume the firm has a **10%** cost of capital. Based on the given cash flows and cost of capital (required return), the NPV can be calculated as follows

Net Present Value (cont.)

Calculation of NPVs for Bennett Company's Capital Expenditure Alternatives

Time lines depicting the cash flows and NPV calculations for projects A and B



Net Present Value (cont.)

	A	B	C
1	**DETERMINING THE NET PRESENT VALUE**		
2	Firm's cost of capital		10%
3		Year-End Cash Flow	
4	Year	Project A	Project B
5	0	$ (42,000)	$ (45,000)
6	1	$ 14,000	$ 28,000
7	2	$ 14,000	$ 12,000
8	3	$ 14,000	$ 10,000
9	4	$ 14,000	$ 10,000
10	5	$ 14,000	$ 10,000
11	NPV	$ 11,071	$ 10,924
12	**Choice of project**		**Project A**

Entry in Cell B11 is
=NPV(C2,B6:B10)+B5
Copy the entry in Cell B11 to Cell C11.
Entry in Cell C12 is IF(B11>C11,B4,C4).

Regents Park Publishers

T1LM4

End