

**Regents Park Publishers**

**Tutorial**



**DEN 423**

**Project  
Management**

# Introduction

- Most realistic projects are large and complex
- Tens of thousands of steps and millions of dollars may be involved
- Managing large-scale, complicated projects effectively is a difficult problem and the stakes are high
- The first step in planning and scheduling a project is to develop the **work breakdown structure**
- Time, cost, resource requirements, predecessors, and people required are identified for each activity
- Then a schedule for the project can be developed

# Introduction

- The program evaluation and review technique (PERT) and the critical path method (CPM) are two popular quantitative analysis techniques to help plan, schedule, monitor, and control projects
- Originally the approaches differed in how they estimated activity times
- PERT used three time estimates to develop a probabilistic estimate of completion time
- CPM was a more deterministic technique
- They have become so similar they are commonly considered one technique, PERT/CPM

# Six Steps of PERT/CPM

1. Define the project and all of its significant activities or tasks
2. Develop the relationships among the activities and decide which activities must precede others
3. Draw the network connecting all of the activities
4. Assign time and/or cost estimates to each activity
5. Compute the longest time path through the network; this is called the **critical path**
6. Use the network to help plan, schedule, monitor, and control the project

The critical path is important since any delay in these activities can delay the completion of the project

# PERT/CPM

- Given the large number of tasks in a project, it is easy to see why the following questions are important
  1. When will the entire project be completed?
  2. What are the **critical** activities or tasks in the project, that is, the ones that will delay the entire project if they are late?
  3. Which are the **non-critical** activities, that is, the ones that can run late without delaying the entire project's completion?
  4. If there are three time estimates, what is the probability that the project will be completed by a specific date?

# PERT/CPM

5. At any particular date, is the project on schedule, behind schedule, or ahead of schedule?
6. On any given date, is the money spent equal to, less than, or greater than the budgeted amount?
7. Are there enough resources available to finish the project on time?

# General Foundry Example of PERT/CPM

- General Foundry, Inc. has long been trying to avoid the expense of installing air pollution control equipment
- The local environmental protection group has recently given the foundry 16 weeks to install a complex air filter system on its main smokestack
- General Foundry was warned that it will be forced to close unless the device is installed in the allotted period
- They want to make sure that installation of the filtering system progresses smoothly and on time

# PERT Chart



# General Foundry Example of PERT/CPM

- Activities and immediate predecessors for General Foundry

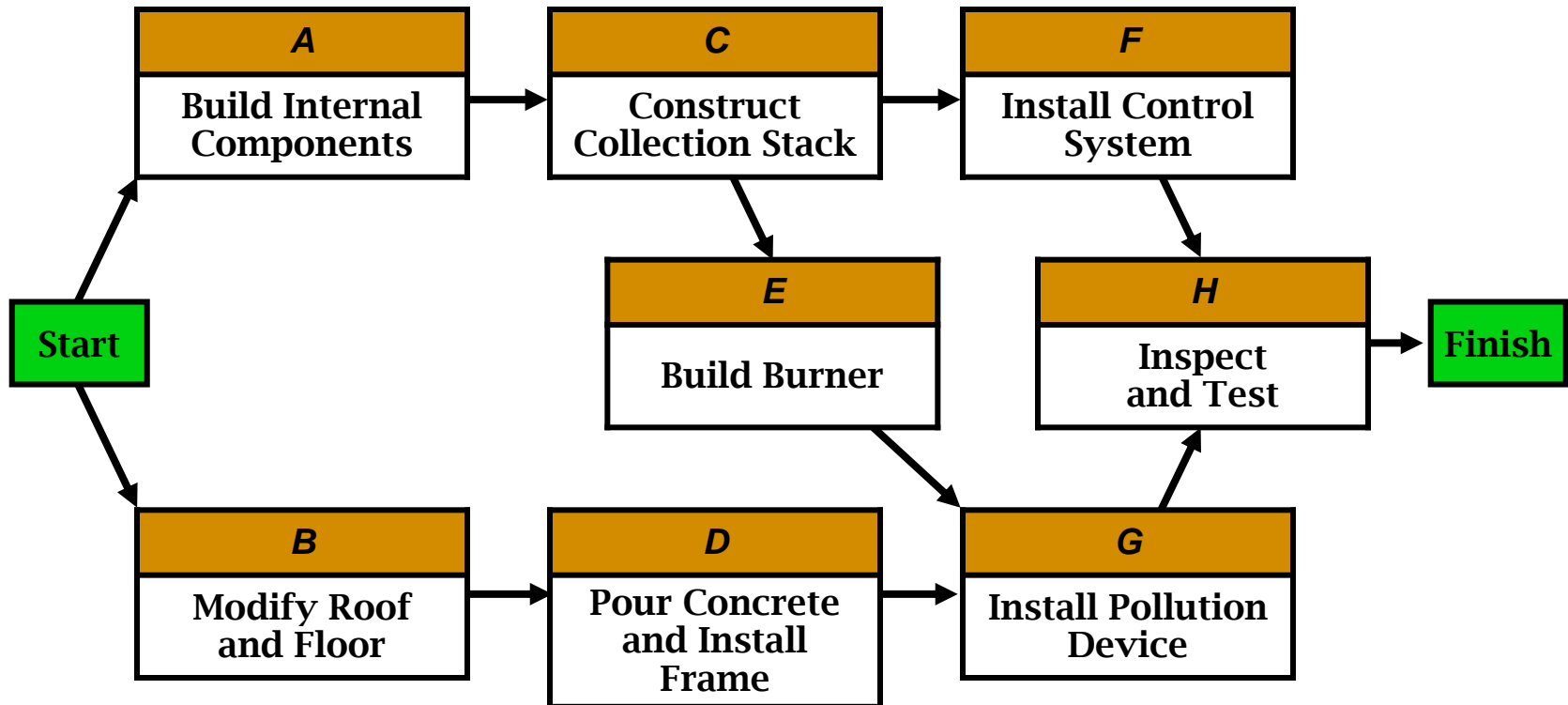
ACTIVITY	DESCRIPTION	IMMEDIATE PREDECESSORS
<i>A</i>	Build internal components	—
<i>B</i>	Modify roof and floor	—
<i>C</i>	Construct collection stack	<i>A</i>
<i>D</i>	Pour concrete and install frame	<i>B</i>
<i>E</i>	Build high-temperature burner	<i>C</i>
<i>F</i>	Install control system	<i>C</i>
<i>G</i>	Install air pollution device	<i>D, E</i>
<i>H</i>	Inspect and test	<i>F, G</i>

# Drawing the PERT/CPM Network

- There are two common techniques for drawing PERT networks
- **Activity-on-node (AON)** where the nodes represent activities
- One node represents the start of the project, one node for the end of the project, and nodes for each of the activities
- The arcs are used to show the predecessors for each activity

# General Foundry Example of PERT/CPM

## ■ Network for General Foundry



# Activity Times

- In some situations, activity times are known with certainty
- CPM assigns just one time estimate to each activity and this is used to find the critical path
- In many projects there is uncertainty about activity times
- PERT employs a probability distribution based on three time estimates for each activity
- A weighted average of these estimates is used for the time estimate and this is used to determine the critical path

# Activity Times

- The time estimates in PERT are

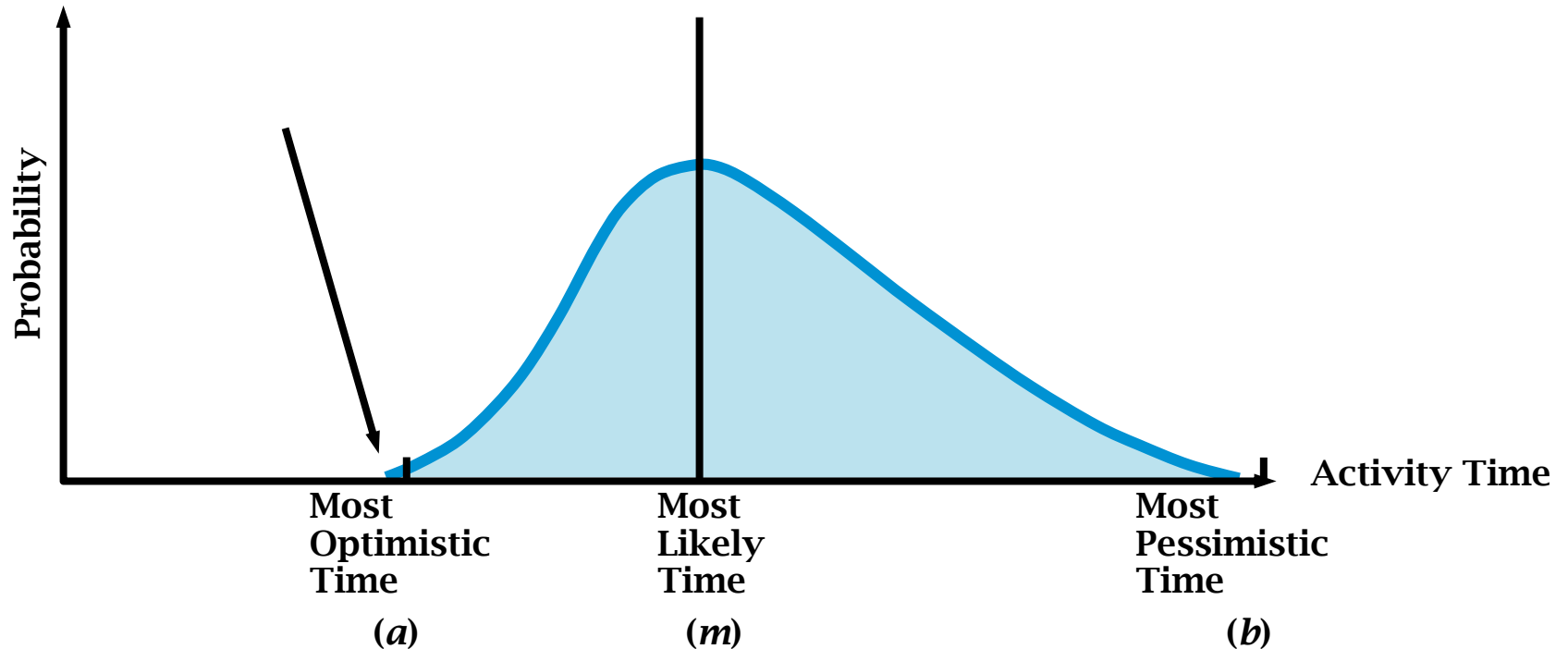
**Optimistic time (a)** = time an activity will take if everything goes as well as possible. There should be only a small probability.

**Pessimistic time (b)** = time an activity would take assuming very unfavorable conditions. There should also be only a small probability that the activity will really take this long.

**Most likely time (m)** = most realistic time estimate to complete the activity

# Activity Times

- PERT often assumes time estimates follow a **beta probability distribution**



# Activity Times

- To find the **expected activity time** ( $t$ ), the beta distribution weights the estimates as follows

$$t = \frac{a + 4m + b}{6}$$

- To compute the dispersion or **variance of activity completion time**, we use the formula

$$\text{Variance} = \left( \frac{b - a}{6} \right)^2$$

# Activity Times Expectations

- Time estimates (weeks) for General Foundry

ACTIVITY	OPTIMISTIC, <i>a</i>	MOST PROBABLE, <i>m</i>	PESSIMISTIC, <i>b</i>	EXPECTED TIME, $t = [(a + 4m + b)/6]$	VARIANCE $[(b - a)/6]^2$
<i>A</i>	1	2	3	2	0.1111
<i>B</i>	2	3	4	3	0.1111
<i>C</i>	1	2	3	2	0.1111
<i>D</i>	2	4	6	4	0.4444
<i>E</i>	1	4	7	4	1.0000
<i>F</i>	1	2	9	3	1.7778
<i>G</i>	3	4	11	5	1.7778
<i>H</i>	1	2	3	2	0.1111
				25	

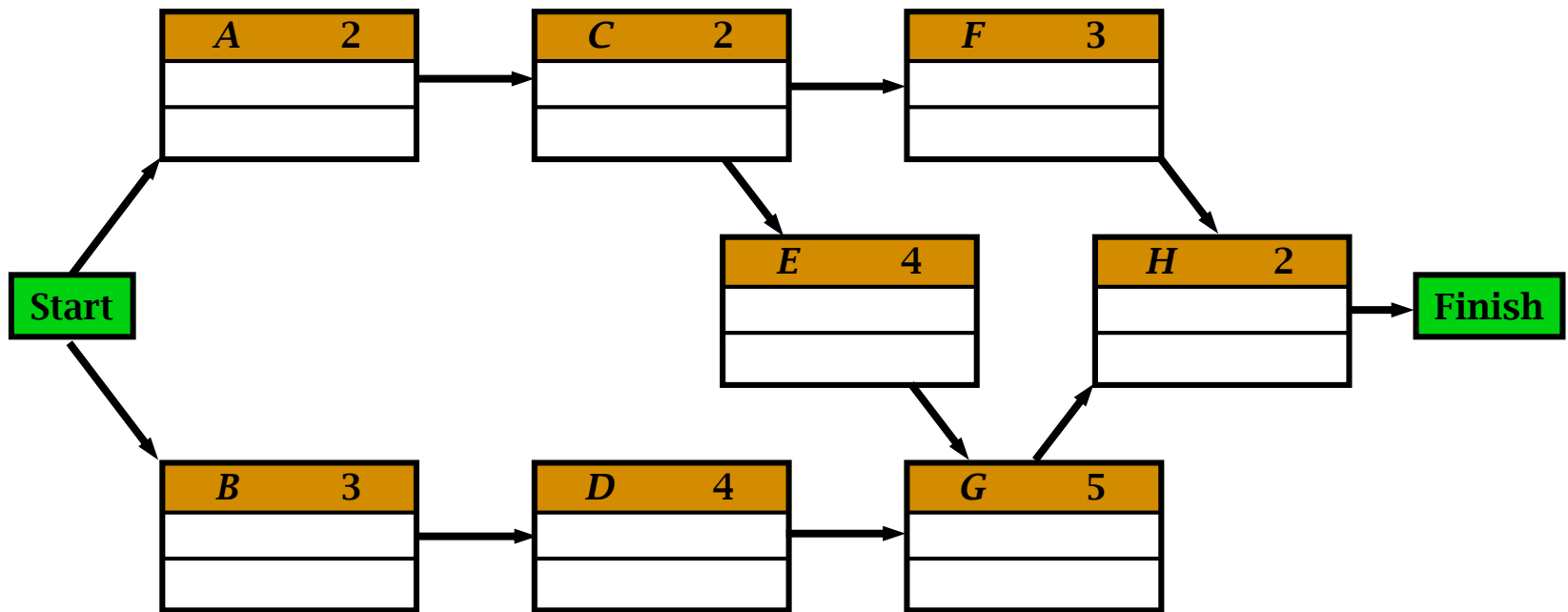


# How to Find the Critical Path

- We accept the expected completion time for each task as the actual time for now
- The total of 25 weeks in does not take into account the obvious fact that some of the tasks could be taking place at the same time
- To find out how long the project will take we perform the critical path analysis for the network
- The **critical path** is the longest path through the network

# How to Find the Critical Path

- General Foundry's network with expected activity times



# How to Find the Critical Path

- To find the critical path, need to determine the following quantities for each activity in the network
  1. **Earliest start time (ES)**: the earliest time an activity can begin without violation of immediate predecessor requirements
  2. **Earliest finish time (EF)**: the earliest time at which an activity can end
  3. **Latest start time (LS)**: the latest time an activity can begin without delaying the entire project
  4. **Latest finish time (LF)**: the latest time an activity can end without delaying the entire project

# How to Find the Critical Path

- In the nodes, the activity time and the early and late start and finish times are represented in the following manner

ACTIVITY	$t$
ES	EF
LS	LF

- Earliest times are computed as

Earliest finish time = Earliest start time + Expected activity time

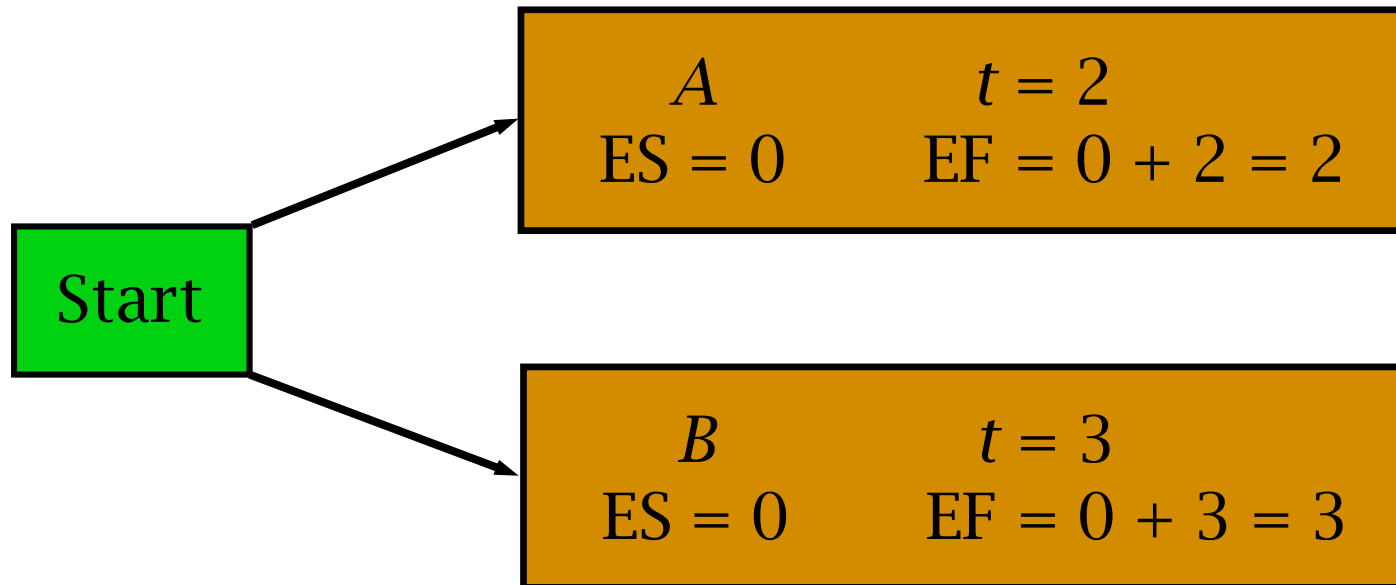
$$EF = ES + t$$

Earliest start = Largest of the earliest finish times of immediate predecessors

ES = Largest EF of immediate predecessors

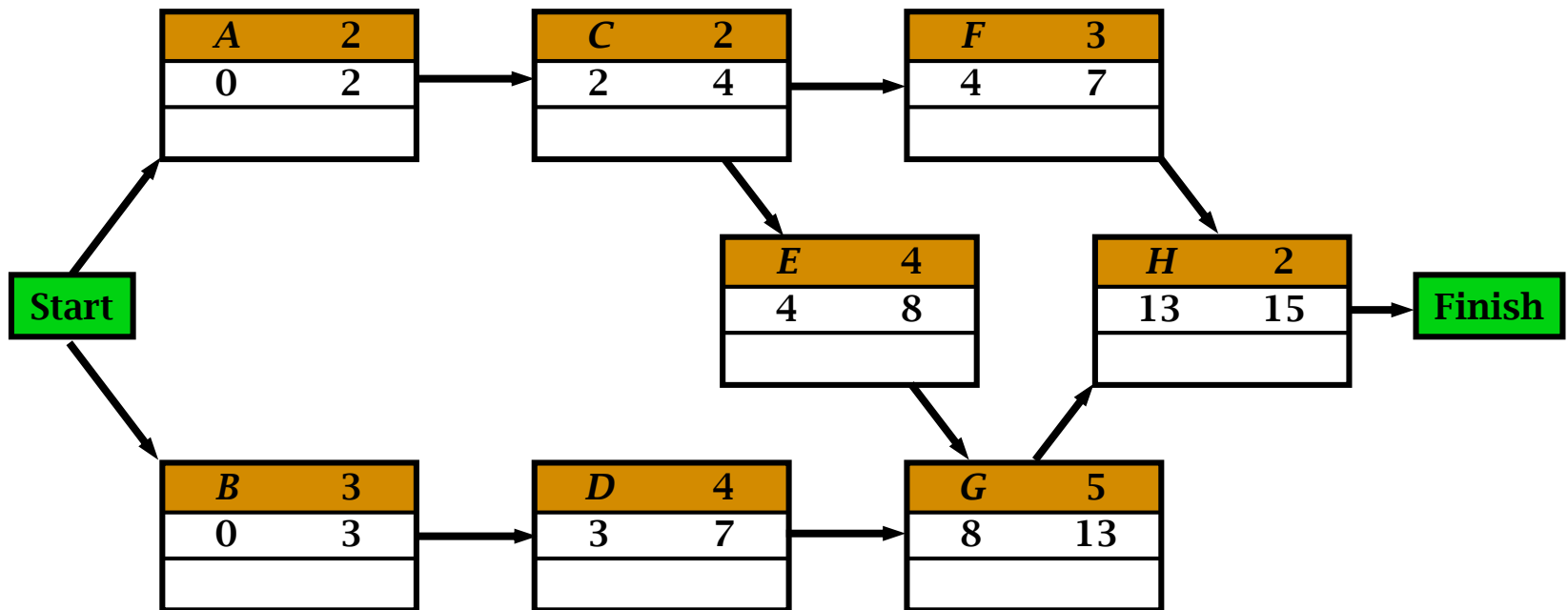
# How to Find the Critical Path

- At the start of the project we set the time to zero
- Thus  $ES = 0$  for both  $A$  and  $B$



# How to Find the Critical Path

- General Foundry's ES and EF times



# How to Find the Critical Path

- Latest times are computed as

Latest start time = Latest finish time  
- Expected activity time

$$LS = LF - t$$

Latest finish time = Smallest of latest start times  
for following activities

LF = Smallest LS of following

activities

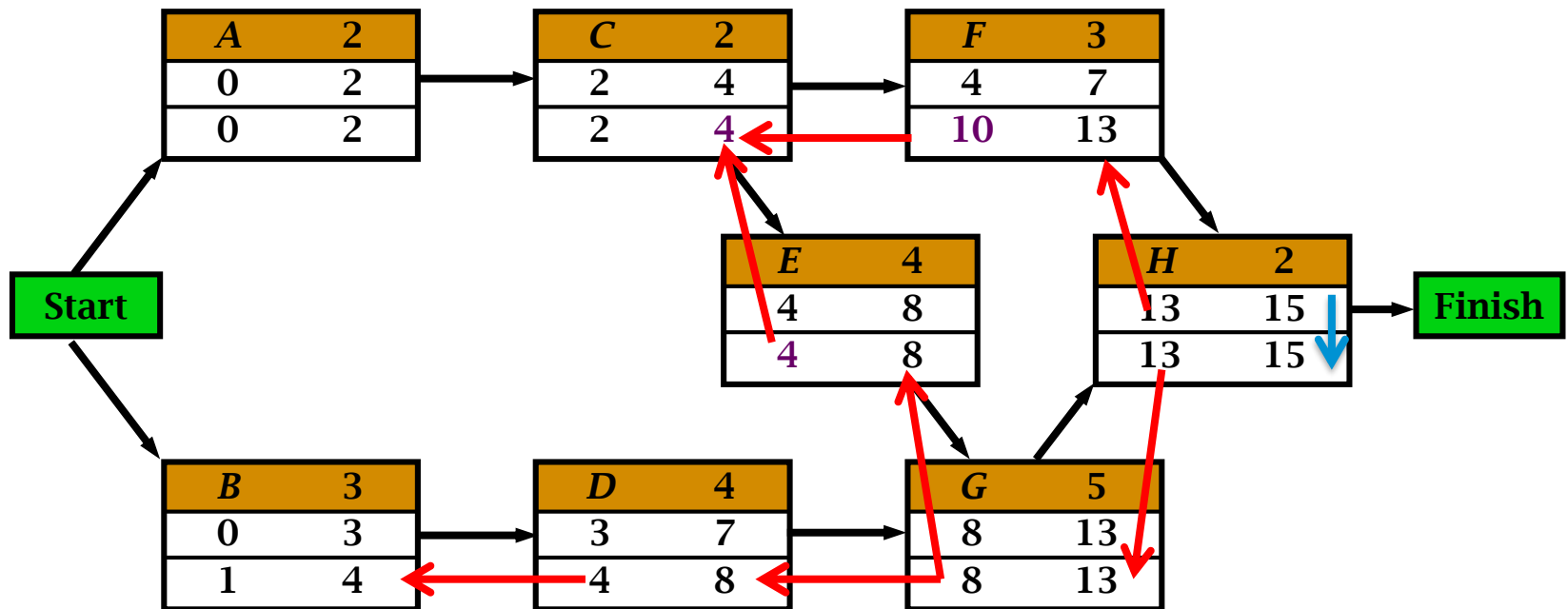
- For activity  $H$

$$LS = LF - t = 15 - 2 = 13 \text{ weeks}$$

# How to Find the Critical Path

- General Foundry's LS and LF times

ACTIVITY	<i>t</i>
ES	EF
LS	LF





# How to Find the Critical Path

- Once ES, LS, EF, and LF have been determined, it is a simple matter to find the amount of **slack time** that each activity has

$$\text{Slack} = \text{LS} - \text{ES}, \text{ or } \text{Slack} = \text{LF} - \text{EF}$$

- We can see activities *A*, *C*, *E*, *G*, and *H* have no slack time
- These are called **critical activities** and they are said to be on the **critical path**
- The total project completion time is 15 weeks
- Industrial managers call this a boundary timetable

# How to Find the Critical Path (alternate solution)

- General Foundry's slack times

ACTIVITY	EARLIEST FINISH, EF	LATEST FINISH, LF	SLACK, LF - EF	ON CRITICAL PATH?
<i>A</i>	2	2	0	Yes
<i>B</i>	3	4	1	No
<i>C</i>	4	4	0	Yes
<i>D</i>	7	8	1	No
<i>E</i>	8	8	0	Yes
<i>F</i>	7	13	6	No
<i>G</i>	13	13	0	Yes
<i>H</i>	15	15	0	Yes

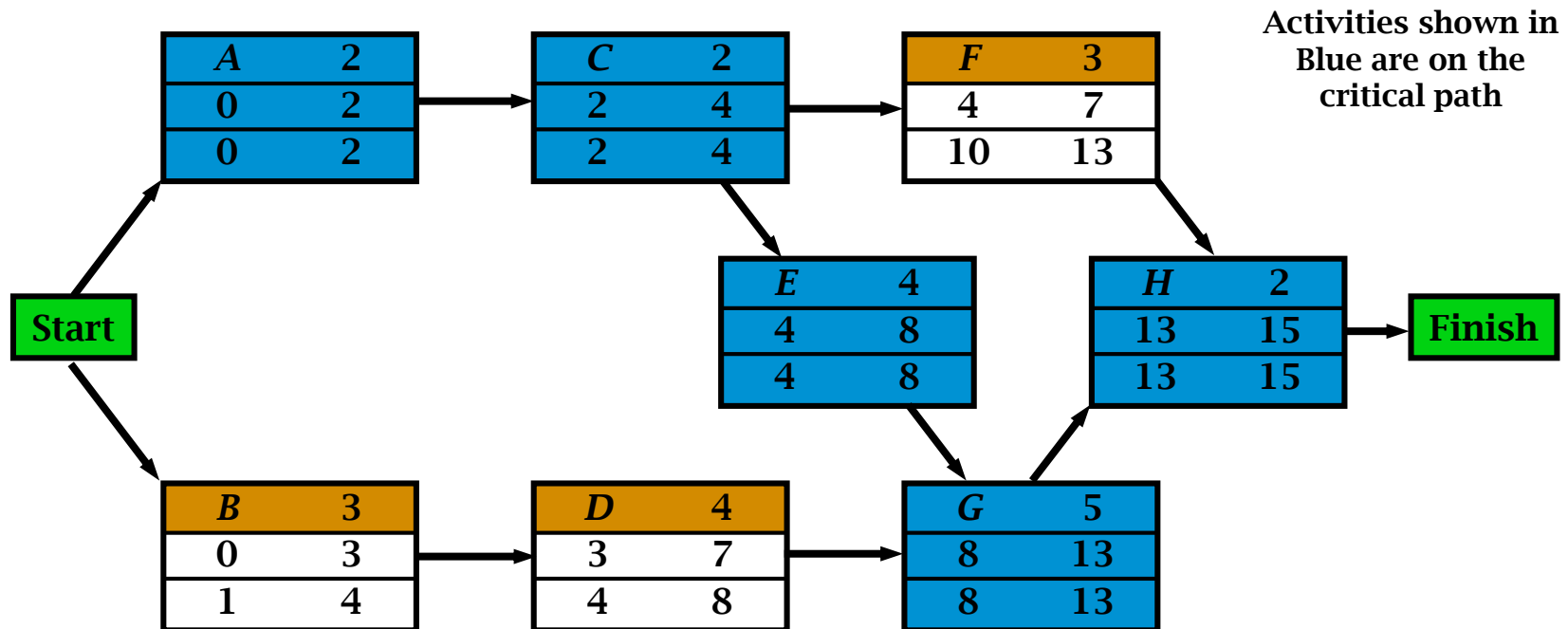
# How to Find the Critical Path

- General Foundry's slack times. Activities with 0 slack times are on the critical path.

ACTIVITY	LATEST START, LS	EARLIEST START, ES	SLACK, Subtract LS - ES	ON CRITICAL PATH?
<i>A</i>	0	0	0	Yes
<i>B</i>	1	0	1	No
<i>C</i>	2	2	0	Yes
<i>D</i>	4	3	1	No
<i>E</i>	4	4	0	Yes
<i>F</i>	10	4	6	No
<i>G</i>	8	8	0	Yes
<i>H</i>	13	13	0	Yes

# How to Find the Critical Path

- General Foundry's critical path



# Probability of Project Completion

- The **critical path analysis** helped determine the expected project completion time of 15 weeks
- But variation in activities on the critical path can affect overall project completion, and this is a major concern
- If the project is not complete in 16 weeks, the foundry will have to close
- PERT uses the variance of critical path activities to help determine the variance of the overall project

$$\text{Project variance} = \sum \text{variances of activities on the critical path}$$

# Probability of Project Completion

We know that

	ACTIVITY	VARIANCE
Activities in Red are on the critical path.	<i>A</i>	<b>0.1111</b>
	<i>B</i>	0.1111
	<i>C</i>	<b>0.1111</b>
	<i>D</i>	0.4444
	<i>E</i>	<b>1.0000</b>
	<i>F</i>	1.7778
	<i>G</i>	<b>1.7778</b>
	<i>H</i>	<b>0.1111</b>

■ Hence, the project variance is

Project variance = **3.111** ←

Add variances shown in red

# Probability of Project Completion

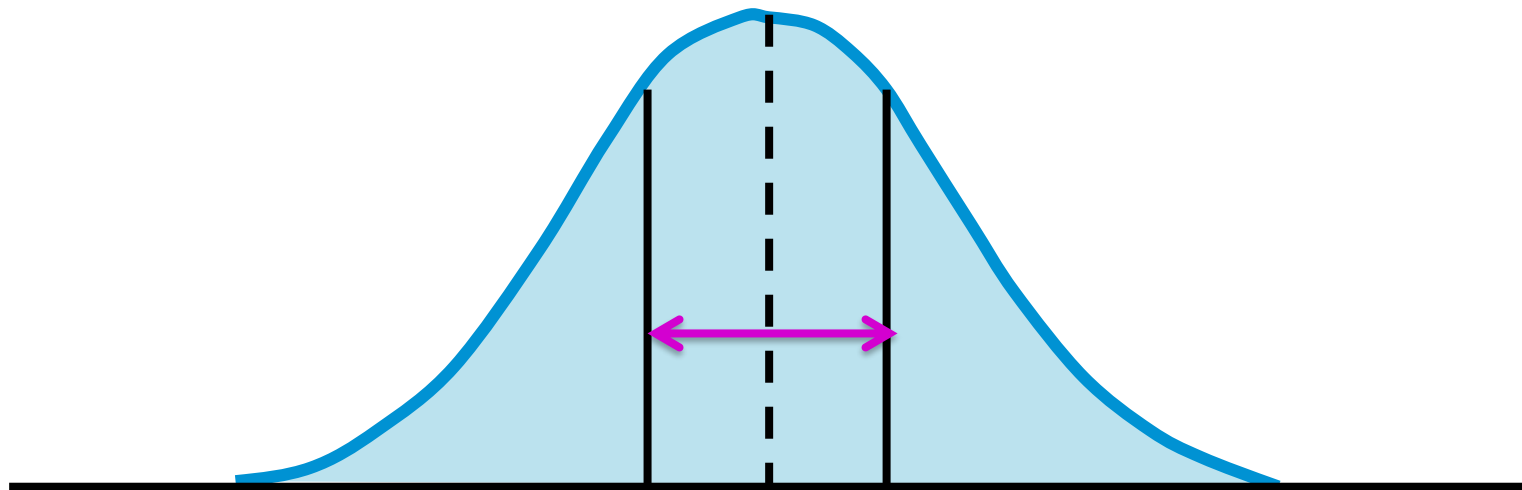
- We know the standard deviation is just the square root of the variance, so

$$\begin{aligned}\text{Project standard deviation} &= \sigma_T = \sqrt{\text{Project variance}} \\ &= \sqrt{3.11} = 1.76 \text{ weeks}\end{aligned}$$

- We assume activity times are independent and total project completion time is normally distributed

# Probability of Project Completion

- Probability distribution for project completion times



**15** Weeks

(Expected Completion Time)

Standard Deviation = +/- **1.76** Weeks



# Probability of Project Completion

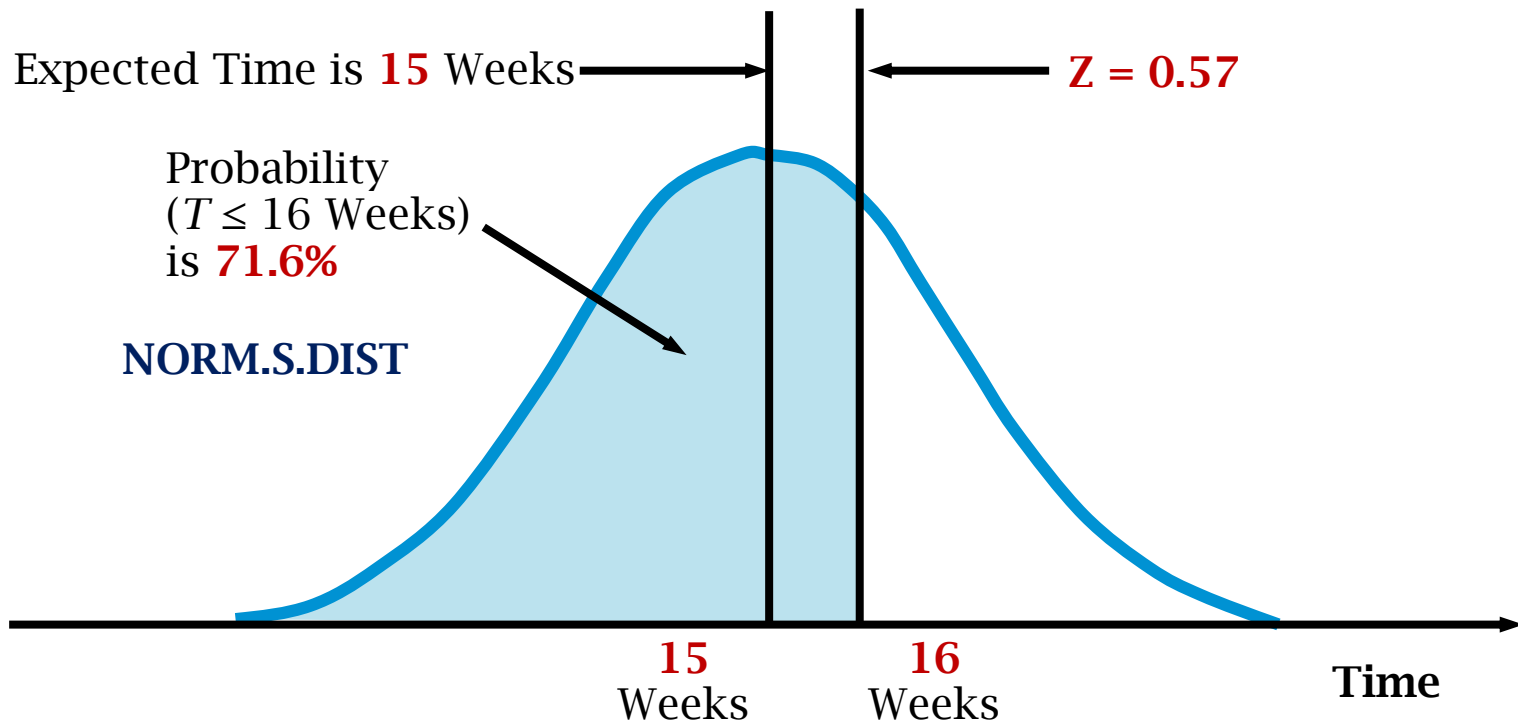
- The standard normal equation can be applied as follows

$$Z = \frac{\text{Due date} - \text{Expected date of completion}}{\sigma_T}$$
$$= \frac{16 \text{ weeks} - 15 \text{ weeks}}{1.76 \text{ weeks}} = 0.57$$

- Using Excel we can find the probability of 0.71566 associated with this Z value
- That means there is a **71.6%** probability this project can be completed in **16** weeks or less

# Probability of Project Completion

- Probability of General Foundry meeting the 16-week deadline

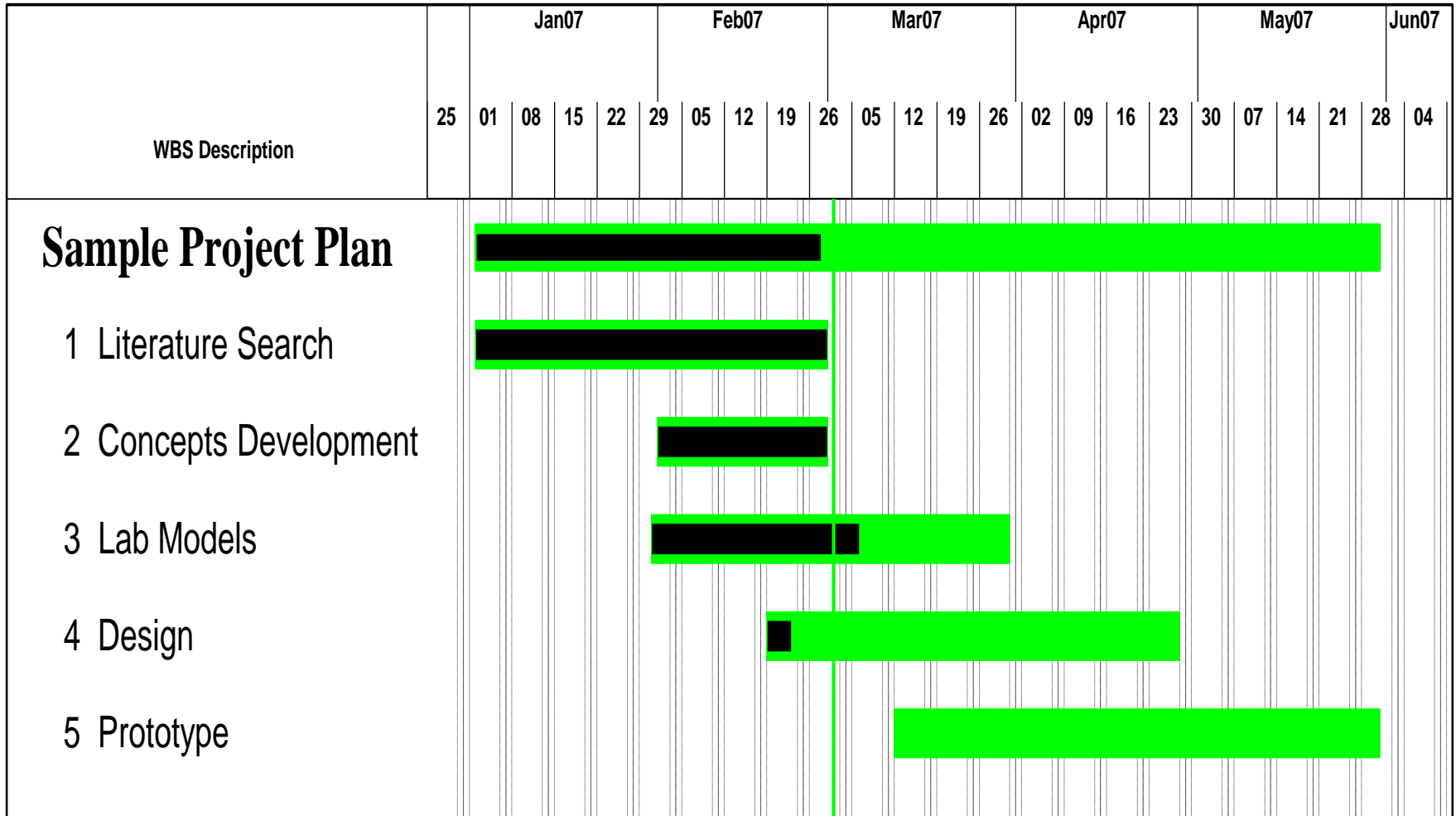


# What PERT Was Able to Provide

- PERT has been able to provide the project manager with several valuable pieces of information
- The project's expected completion date is 15 weeks
- There is a 71.6% chance that the equipment will be in place within the 16-week deadline
- Five activities (*A, C, E, G, H*) are on the critical path
- Three activities (*B, D, F*) are not critical but have some slack time built in
- A detailed schedule of activity starting and ending dates has been made available

# Gantt Chart

# Gantt Chart



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# **Project Management**

**End**