CEN 421 Week 4 Lecture Notes

These lecture notes are meant to be a complement to the textbook. It is essential that the student reads the material in the textbook.

Time Value of Money

We know that there is a time value of money. If you have the choice of receiving a dollar today or receiving a dollar in one year, it is better to get the money today. At the least, you could put it in the bank and assuming five percent interest, you would have $1.05 in one year.

The time value of money is at the heart of all financial calculations. The value of any asset; stocks, bonds, apartment houses, or business equipment is the present value of the future cash flows. These cash flows have to be brought back to the present at some interest rate (the time value of n=money). We call this process discounting future cash flows.

Calculations

There are various formulas for calculating present value, future value. etc. There are also tables for some interest rates and time periods. There are inexpensive financial calculators that will do all of these calculations. The textbook gives the keystrokes for the TI BAII. I prefer the HP12C and all of the examples in these notes will refer to the HP12C. You can also go on the internet to www.hp.com and type in hp12c for an online tutorial. You will notice that on your financial calculator there are five variables (n, I, PV, PMT, FV).

'n' is the number of compounding periods. This is usually in years, but it is actually the number compounding periods like months for a loan that is paid monthly.

'i' is the interest rate. This is usually the annual or yearly rate. However, if you are compounding monthly, the rate (i) would be the annual rate divided by 12.

'PV' is the present value.

'PMT' is the periodic (annual or monthly) payment.

'FV' is the future value.

If you know any four of these five variables, you can find the fifth. Often, we are only using for of these. In that case, put a zero in the one not needed. For example, we have a present value an interest rate and a number of years and we are looking for the future value. You put a zero in for PMT.

It is very important to keep you signs straight. Money going out is always negative and money coming in is always positive.

It is also very important to be consistent with the number of periods (n) and the interest rate (i). For monthly payments, like your home or car payments, multiply the number years by 12 and divide the annual interest rate by 12.

The calculator has continuous memory. It is best to always clear the calculator before starting each problem.

Future Value

Assume that we have a set amount of money and we want to know its future value. We also know the interest rate (or compounding rate) and the number of years. We will solve example 15-1 in the text using the HP12C. We have $1,000 present value, and five years at 10%. Here are the keystrokes:

enter 1000 [CHS] PV
enter 5 n
enter 10 i
find FV 1,610.51 (solution)

Here is the solution for example 15-2:

enter 1000 [CHS] PV
enter 12 n (number of periods)
enter 10 i (interest rate)
find FV 1,126.83 (solution)

Rule of 72

There is a very simple "rule of thumb" to determine how long it will take to double your money. Divide 72 by the interest rate and the result will be the number of years to double. For example, at 10%, how long will it take? (72/10 =7.2 years) Your money will double in about 7.2 years at 10%.

We can also determine at what interest rate we need to double. For example in 8 years it will take about 9 years (72/8 = 9). You would need to earn 8% to double your money in 9 years.

Present Value

We can find the present value of a future amount by using our calculator. Here is the solution to example 15-3. Determine the present value of $1,000 at 10%, five years from now.

Enter 1,000 F
Enter 5 n
Enter 10 i
Find PV -620.92 (the solution)

Uniform Series to Find a Future Value

When the payment is uniform for some period of time (these are also called annuities), we can use the payment key (PMT) on the calculator. The payment amounts must all be the same to do this.

Usually, payments are made at the end of the compounding period. When you make your house payment on August 1, it was actually for the month of July. We call this payment in arrears or an ordinary annuity. Sometimes payments are made at the beginning of the period. If you are renting, the payment on August 1, is for the month of August. We call this an annuity due.

For an ordinary annuity, set you calculator on 'end'. This is the default setting on your calculator. For an annuity due, set you calculator on 'begin'. This will ensure that the payment is made at correct time.

We will work example 15-5. Determine the future value, ten years from now, of $500 paid at the end of each year (ordinary annuity) at 9%.

Enter 500 [CHS] PMT
Enter 10 n
Enter 9 i
Find FV 7,596.46 (the solution)

Uniform Series to Meet a Future Value

To find the annual amount needed to meet a future value, we do a similar operation, only this time we know the future value and we are looking for the payment.

We will work example 15-6. Determine the cash flows needed to have $100,000 in 20 years if our money will compound at 10%. This is also called a sinking fund.

Enter 100,000 FV
Enter 20
Enter 10 i
Find PMT -1,745.96 (the solution)

Uniform Series to Find a Present Value

This time we know the payment series amount and we are looking for the present value. We will work example 15-7 where we the annual cash flow is $500 for 5 years at 6%.

Enter 500 PMT
Enter 5 n
Enter 6 i
Find PV -2,106.18 (the solution)

Uniform Series – Capital Recovery

Now we want to find how much our payments will be if we know the present value, the number of years and the interest rate. This is also called capital recovery. We will work example 15-8 where the present value $10,000, the time is 5 years and the interest rate is 1% per month. This problem is compounded and paid monthly.

Enter 10,000 [CHS] PV
Enter 1 i
Enter 60 n (5x12)
Find PMT 222.44 (the solution)

This process is call amortizing a loan. Each payment includes both principal and interest. When the last payment is made, the loan is paid off.

This is process used on mortgages for homes. For example, take a $300,000 at 6.5% for 30 years.

Enter 300,000 PV
Enter 0.5417 i (6.5/12)
Enter 360 n (30x12)
Find PMT -1,896.20 (the monthly payment)

Uneven Cash Flows

Using the payment key (PMT) only works when all of the periodic payments are equal. When they are not equal, we need to use the cash flow keys.

It is often useful to draw a cash flow diagram. The horizontal axis shows the time, usually in years. The cash flow arrows are up for positive cash flow (money coming in) and down for negative cash flow (money going out).

For example 15-9, we can use the normal financial keys since all of the payments are equal.

Enter 2000 PV
Enter 1000 PMT
Enter 10 n
Enter 7 i
Find FV -17,750.75 (solution)

For example 15-10, you will need the cash flow keys. CFo (cash flow zero) is in blue under the PV key. CFj (all other cash flows) is in blue under the PMT key. In this example, we are looking for the future value of an unequal cash flow stream. We have a net present value key (NPV) key, but not a net future value key. Therefore, we will have to solve this in two steps. First find the net present value and then the future value.

Step 1

Enter 0 [g] CFo
Enter 10,000 [g] CFj
Enter 15,000 [g] CFj
Enter 25,000 [g] CFj
Enter 21,000 [g] CFj
Enter 17,000 [g] CFj
Enter 1.5 I (the monthly interest rate)
Find [f] NPV 83,888.36 (the net present value)

Step 2

Enter 83,888.36 PV
Enter 1.5 i
Enter 6 n (number of months)
Find FV -91,724.98 (our answer)

To solve example 15-11, we only need one step.

Enter 0 [g] CFo
Enter 1000 [g] CFj
Enter 1000 [g] CFj
Enter 2000 [g] CFj
Enter 1000 [g] CFj
Enter 1000 [g] CFj
Enter 12 i
Find [f] NPV 4,316.56

The net present value (NPV) is the present equivalent of all the future cash flows discounted back to the present at some interest rate. This interest rate (often

called the discount rate) is usually our cost of money. That is what it costs us to borrow money or the rate we could invest the money at.

Finding the Interest Rate

Remember, if you know any four of the five financial variables, you can find the fifth. In example 15-13, we know the present value, the future value, and the number of years. We always need to make one of values negative and the payment equal to zero.

Enter 1,000 [CHS] PV
Enter 1,177.22 FV
Enter 2 n
Find i 8.5 (8.5%)

In example 15-14, we know the present value and the payments. We still have to make one negative and the future value equal to zero.

Enter 1,000 PV
Enter 550 [CHS] PMT
Enter 2 n
Find I 6.6 (6.6%)

For example 15-15, we will need the cash flow keys. The solution will be the internal rate of return (IRR). The IRR is the interest rate that makes the futures cash flows equal to the present value. That is the NPV is equal to zero.

Enter 1000 [g] CFo
Enter 500 [CHS] [g] CFj
Enter 600 [CHS] [g] CFj
Find [f] IRR 6.39 (the interest rate or the internal rate of return)

Inflation

Inflation is a fact of life. Since money is only backed by the faith of the government (not gold), the only limit on the growth of the money supply is the will of the government (not printing more money). In the United States, the Federal Reserve System has been given the authority to control inflation. In the past 20 years they have done a good job and the inflation rate is now about 3%. Inflation is the supply of money growing faster than the economy. In many developing countries, the government does not have strong controls and the inflation rate is very high.

The easiest way to calculate historic inflation is to use your calculator. For example, an item cost $1.00 nine years ago and now it costs $2.00, we can determine the average inflation rate for the past nine years.

Enter 1.00 [CHS] PV
Enter 2.00 FV
Enter 9 n
Find i 8.01 (the average annual inflation rate)

The interest rate you receive on your money is the nominal rate interest rate. The real interest rate is the nominal rate less the inflation rate. The formula in the textbook is the most accurate method of determining the nominal rate if you

know the real rate and the inflation rate. However, it is usually good enough to just add the expected future inflation rate to the real rate to get the nominal rate.

IRR and NPV

Most companies, including construction companies, use a discounted cash flow (DCF) process to analyze new projects and new ventures. The two most common tools of analysis are the net present value (NPV) and the internal rate of return (IRR).
If the NPV is positive, you are adding value to the company. If the IRR is greater than the cost of funds, you are adding value to the company.

Annual Percentage Yield and Effective Annual Rate
Interest rates are often compounded more often than once a year (annually). We need to be able to find the effective annual rate (EAR) in order to get a comparison between loans with different compounding periods.
The EAR is found with the following formula:
$EAR = (1 + i/n)^n - 1$, where (i) is the nominal annual rate and (n) is the number of times it is compounded annually. For example, a 12% rate compounded monthly would be:

$EAR = (1 + 0.12/12)^{12} - 1 = 0.1268 = 12.68\%$

The more often that the interest is compounded, the larger the EAR will be.

Loans

Loans can be either fixed rate of variable rate loans. With a fixed rate loan, the bank takes the risk if interest rates increase in future years. With a variable rate loan, often called adjustable rate mortgages (ARMS), the borrower takes the risk of increasing rates. An important factor to the borrower is the length of time he expects to keep the loan. If you plan to stay in your property for 20-30 years, then get a fixed rate loan. If you will only be there five years, get a variable rate loan.
We will work example 16-9 with the HP12C.

Enter 150,000 PV
Enter 9 [g] i (the monthly rate)
Enter 30 [g] n (the number of months)
Find PMT -1,1206.93 (the monthly payment)

Now we will solve example 16-11 where we are looking for the first month's interest. Leave everything in your calculator from example 16-9.

Enter 1 [f] AMORT
Get -1,125.00 (the first month's interest)
Enter x=y
Get -81.93 (the first month's principal)

Note that the interest (1,125.00) and the principal (81.93) have to add up to the payment amount (1,1206.93).
The entire amortization schedule can be developed be repeating this process (1[f]AMORT).

To find the payoff balance at any on your loan, we will work example 16-12. This is also called a balloon payment. First set up example 16-9 again.

Enter 15 [g] n (180 months)
Find FV -118,995.73 (the loan balance in 15 years)

Closing Costs

Closing costs are the origination costs of the loan. These costs include the appraisal, title insurance, loan commission, etc. We can use our calculator to calculate to actual rate you will be paying on the loan, not the stated rate. In example 16-14, we have a $100,000 loan with $2,500 in closing costs. This really means that we have to repay $100,000, but only receive $97,500.

Enter 100,000 PV
Enter 9 [g] i
Enter 30 [g] n
Find PMT -804.62 (the monthly payment)
Enter 97,55 PV
Find i 0.7738
Enter 12 x
Get 9..29% (the APR)

To get the EAR = $(1 + 0.007738/12)^{12} - 1 = 0.0969 = 9.69\%$

Discount Loans

A discount loan is where the interest is paid up front (also called prepaid interest). This is a great deal for the bank, but not a good deal for the borrower. We will do example 16-17 using our HP12C. The interest rate is 10% on a $100,000, one-year loan paid in advance.

Enter 90,000 PV
Enter 100,000 [CHS] FV
Enter 1 n
Find i 11.11%

Leasing

Leasing is another way to finance equipment. It is important to know that a lease is a similar obligation to a loan. Once you sign a lease, you cannot back out. With you calculator, you can make a comparison analysis to determine if it is more economical to buy or lease.

Financial Decisions

When a company (or an individual for that matter) needs to make a financial decision, we have some tools at our disposal.
All relevant income and costs due to the decision should be used.
Sunk costs should not be included. Since this money has already been spent, it will not have any bearing on your analysis.

Required Rate of Return

The required rate of return needs some consideration. The return you can get on a safe investment like a CD needs to be considered. The safety or risk of the investment and the length of time also need to be considered. For a company, the required rate is the return on their investment that the stockholders and bondholders expect.

Making the Financial Decision

As I mentioned earlier in these notes, most companies use the net present value (NPV) and internal rate of return (IRR) methods.

I personally prefer to use the modified internal rate of return (MIRR) method. With the MIRR, the positive cash flows are carried forward at some reasonable reinvestment rate. Then you have a present value (the initial investment) and a future value (the sum of the cash flows carried forward at the reinvestment rate). With you calculator, you have the (PV), the (FV), and the (n). Now solve for (i). This is the MIRR.

The payback period method is simple and useful for small investment decisions. It basically tells you when you will have recovered your investment. You probably would not use this method on big decisions, but it is fine for small ones.

Uncertainty

We have been discussing making financial decisions that involve future estimated cash flows. Anytime we make future projections, there is uncertainty. The fact is, we really do know what will happen, we are making our best estimate.

It is useful to perform a scenario analysis. Use your best estimate as the most probable case. Now assume everything goes right and do a best-case analysis. Next, assume everything goes wrong and do a worst-case analysis (remember Murphy's Law that says 'anything that can go wrong will go wrong'). There are many things that you cannot control that will affect your project and your company. Things like inflation, interest rate hikes, competition, taxes, recessions, and government regulations are completely out of your control.

Taxes

Taxes can be a major expense for the company. All of the cash flows that we use in financial analysis should be after tax (ATCF). This includes all the cash flows in net present value and internal rate of return calculations.

You should always take tax deductions as soon as possible, remember, the time value of money.

Taxes may make the decision to lease or buy equipment. There are tax implications to both methods.