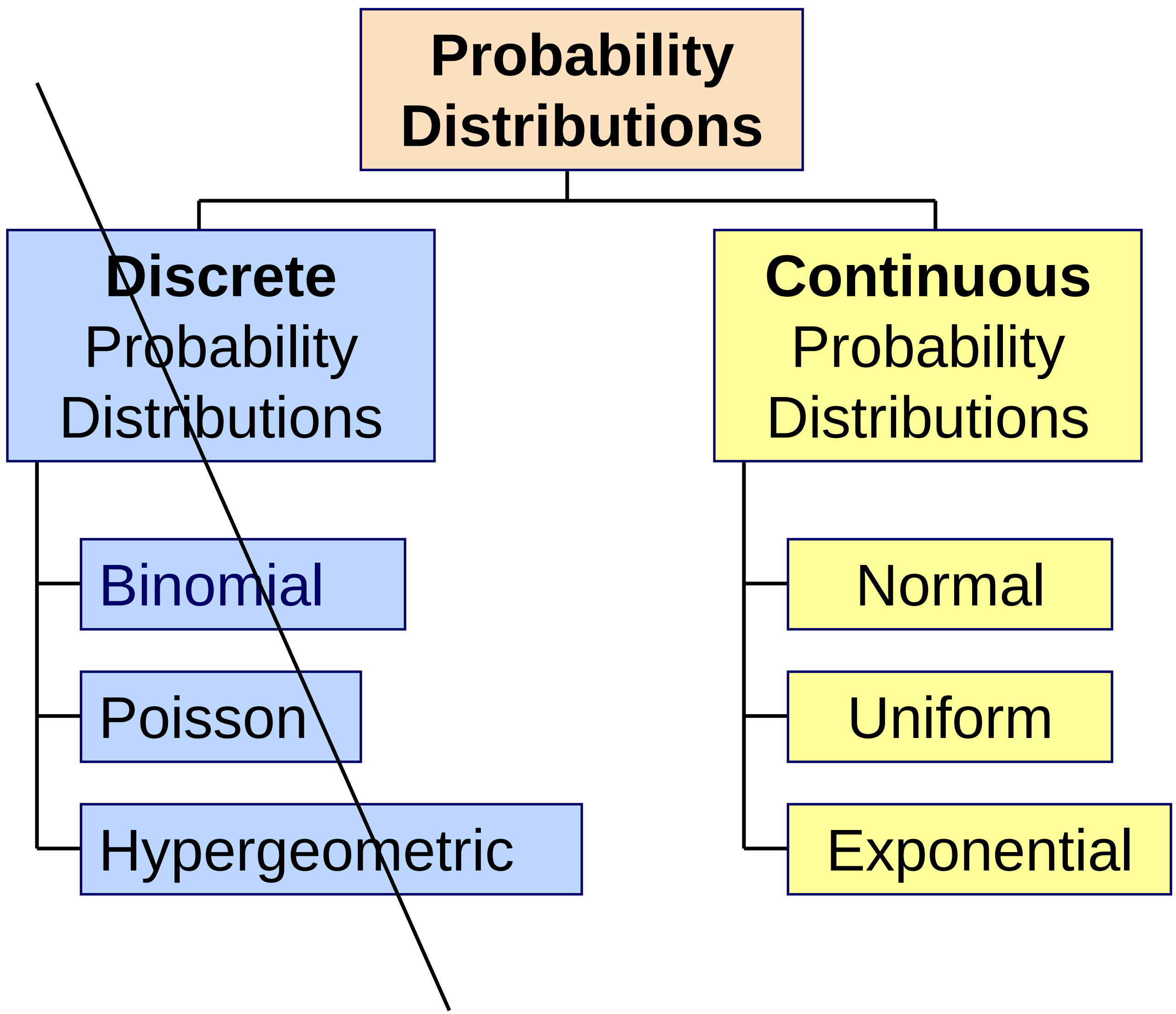
Continuous Probability Distributions

Chapter Goals

After completing this chapter, you should be able to:

- Apply the binomial distribution to applied problems
- Compute probabilities for the Poisson and hypergeometric distributions
- Find probabilities using a normal distribution table and apply the normal distribution to business problems
- Recognize when to apply the uniform and exponential distributions

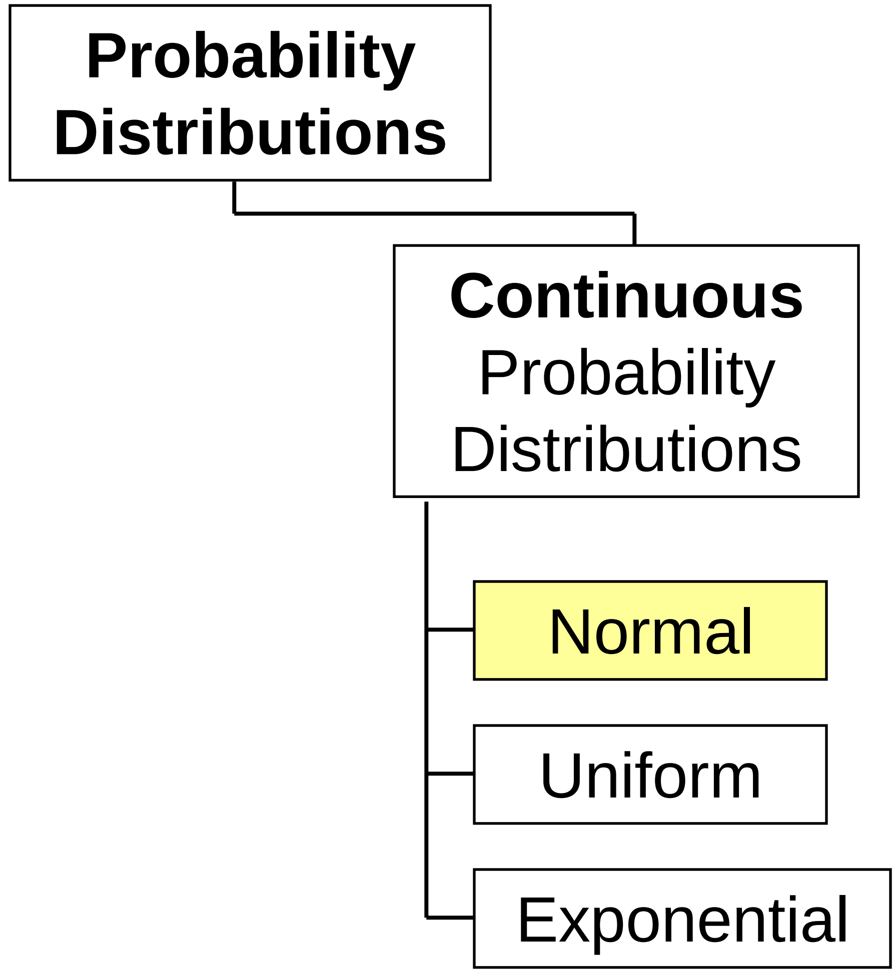
Probability Distributions



Continuous Probability Distributions

- A continuous random variable is a variable that can assume any value on a continuum (can assume an uncountable number of values)
  - thickness of an item
  - time required to complete a task
  - temperature of a solution
  - height, in inches
- These can potentially take on any value, depending only on the ability to measure accurately.

The Normal Distribution



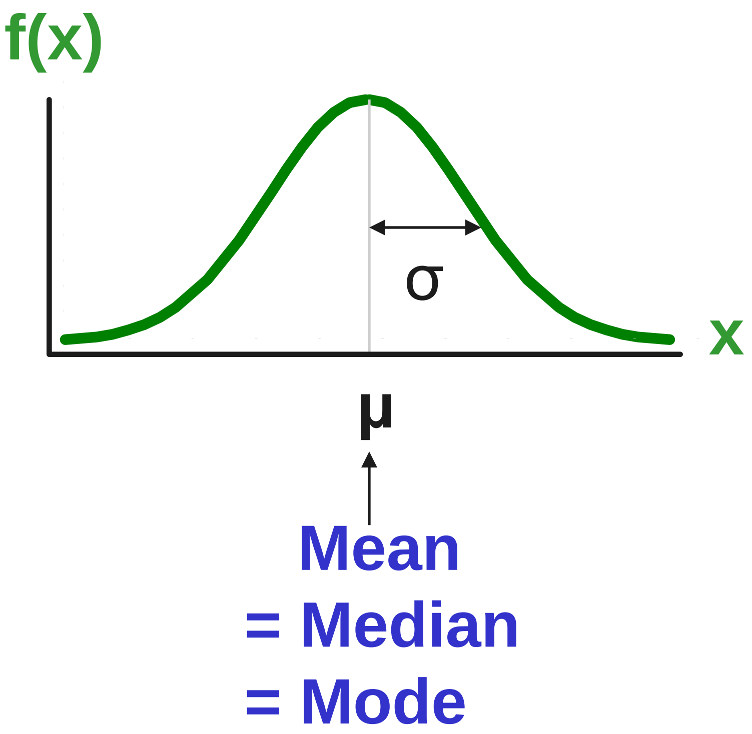
The Normal Distribution

- Bell Shaped
- Symmetrical
- Mean, Median and Mode are Equal

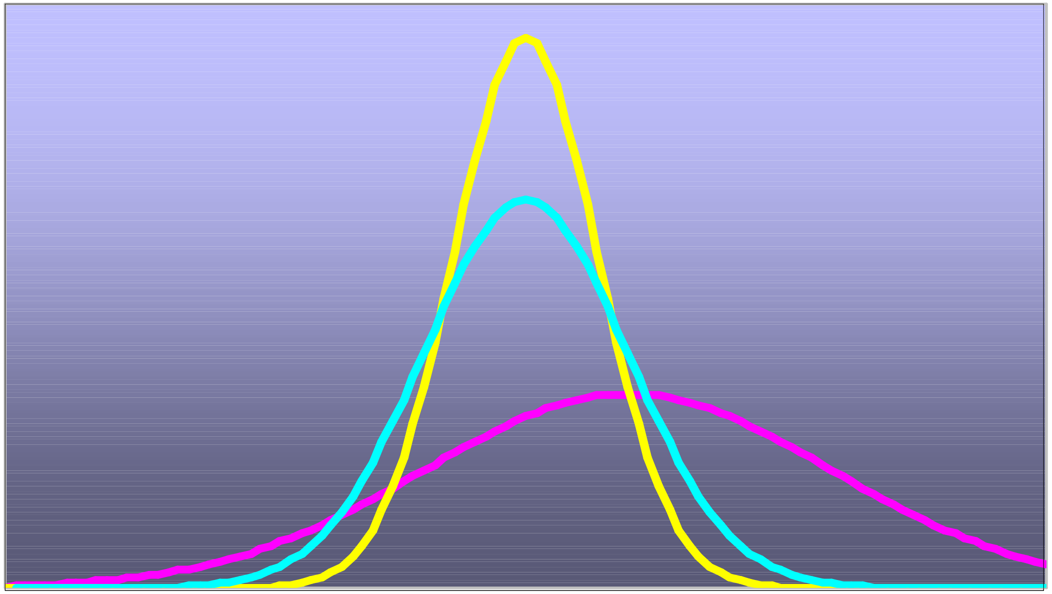
Location is determined by the mean, μ

Spread is determined by the standard deviation, σ

The random variable has an infinite theoretical range: + ∞ to − ∞

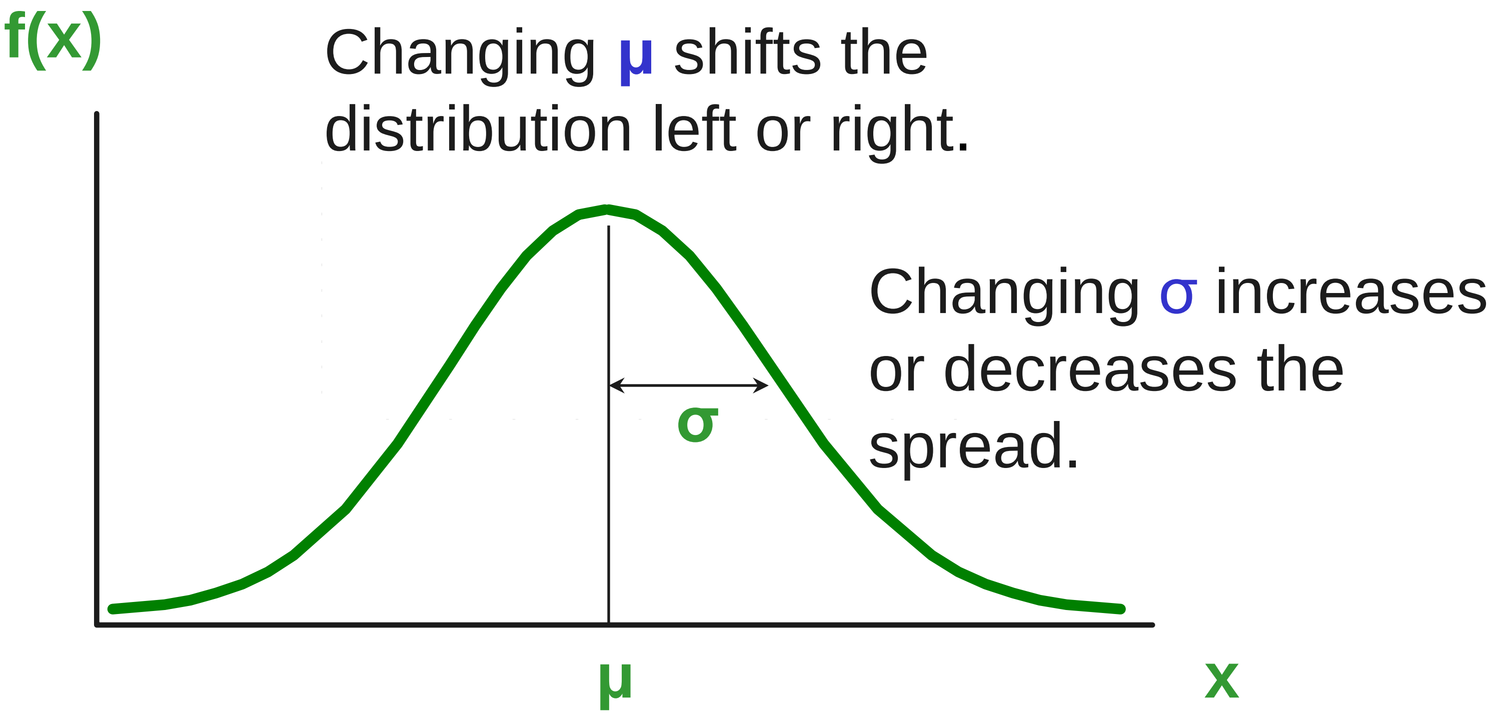


Many Normal Distributions



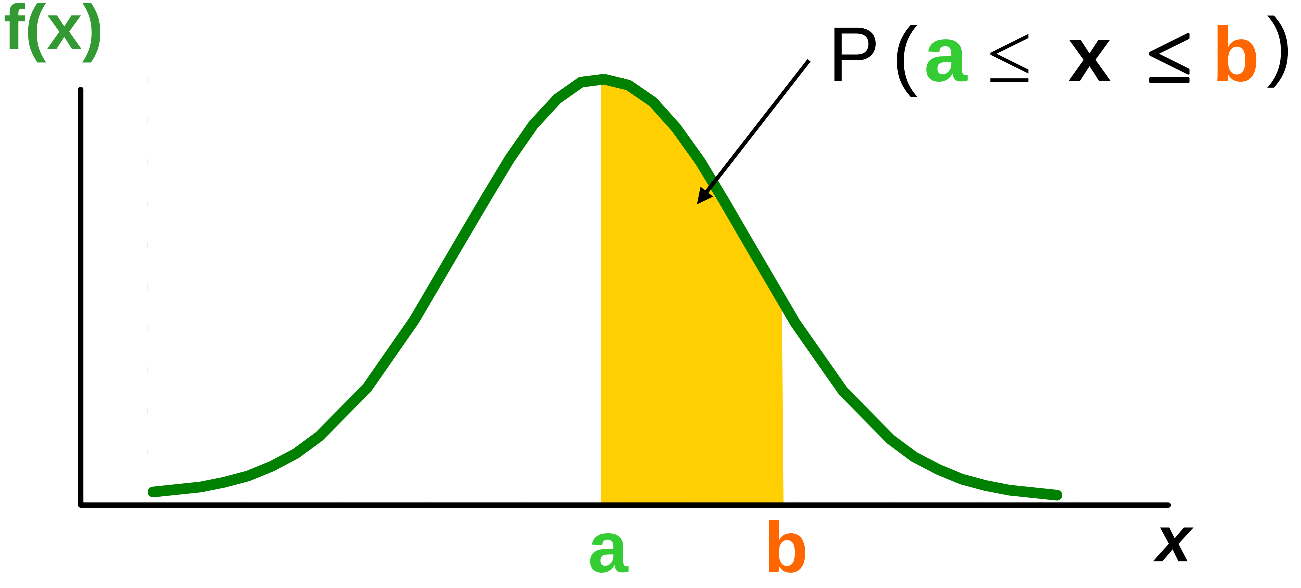
By varying the parameters μ and σ, we obtain different normal distributions

The Normal Distribution Shape



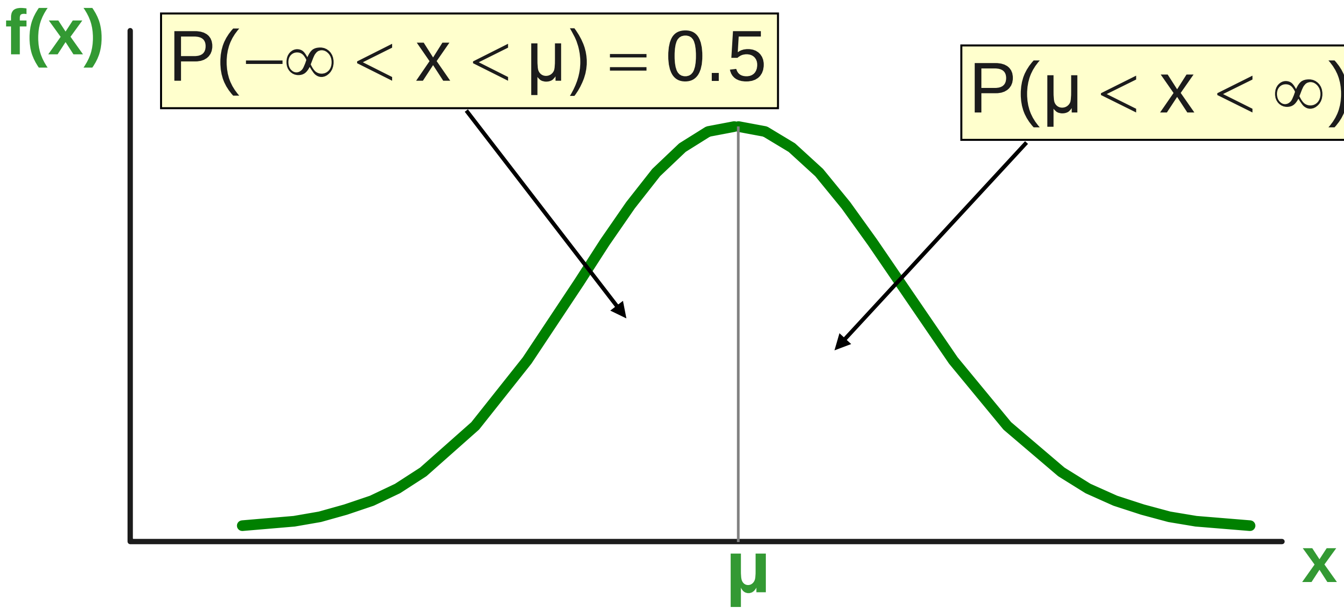
Finding Normal Probabilities

Probability is measured by the area under the curve



Probability as Area Under the Curve

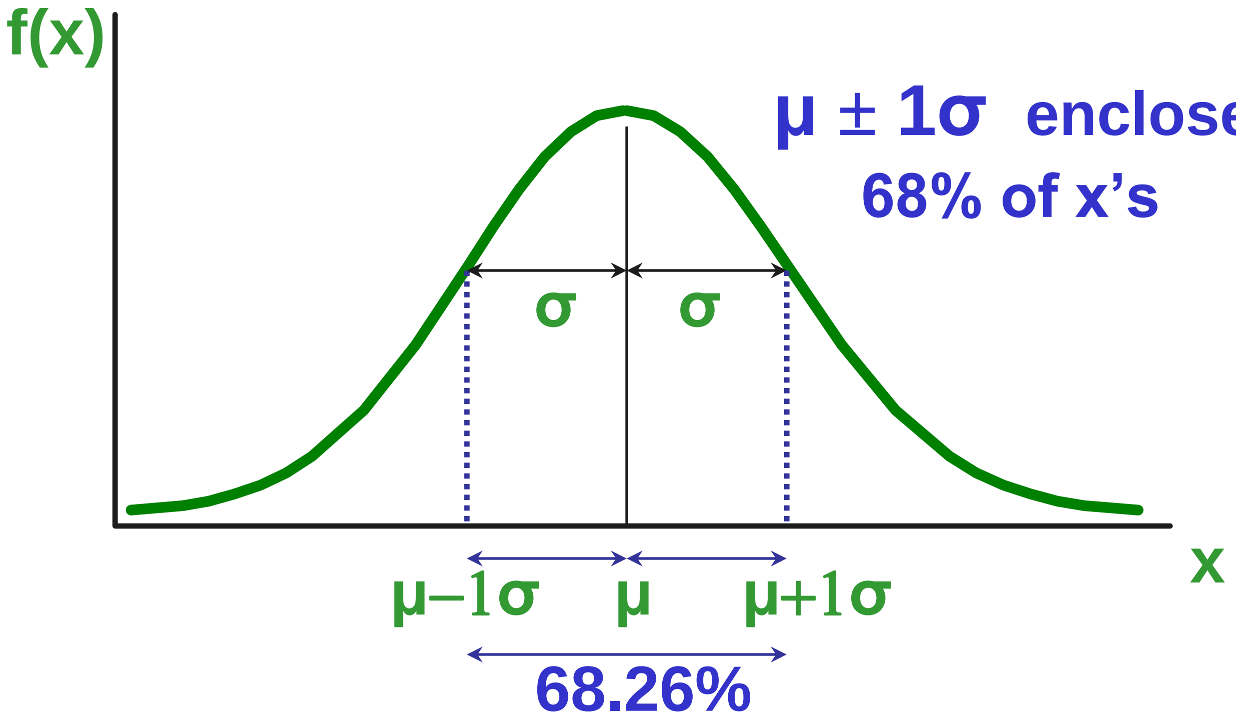
The total area under the curve is 1.0, and the curve is symmetric, so half is above the mean, half is below



$$P(-\infty < x < \infty) = 1.0$$

Empirical Rules

What can we say about the distribution of values around the mean? There are some general rules:



The Empirical Rule

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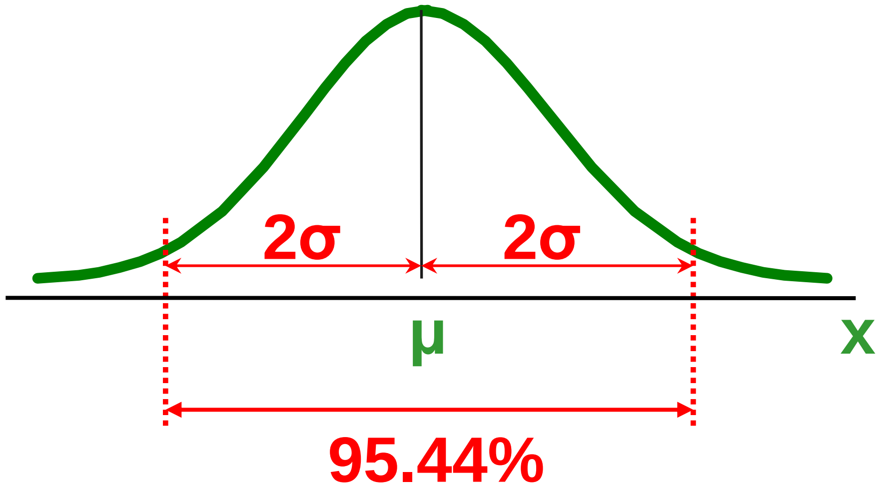
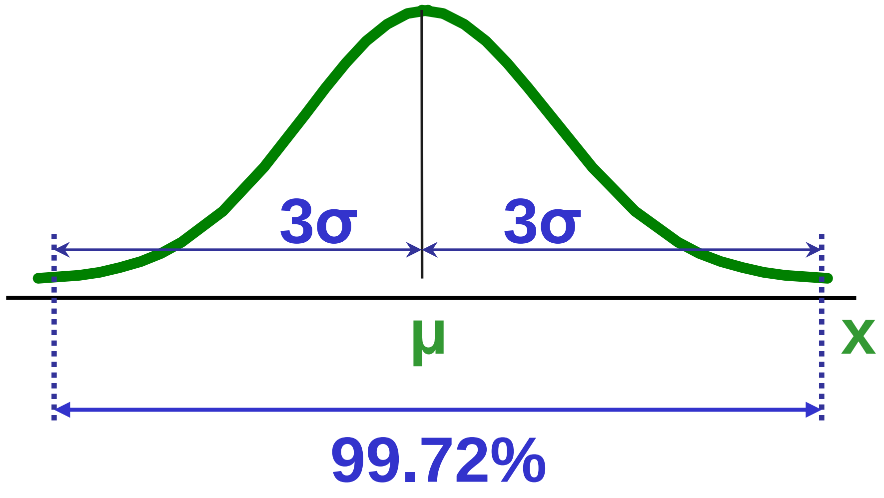
- $\mu \pm 2\sigma$ covers about 95.44% of x's
- $\mu \pm 3\sigma$ covers about 99.72% of x's





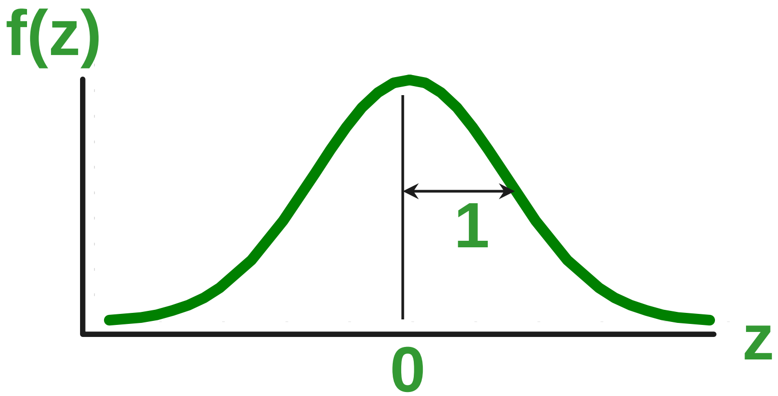
Importance of the Rule

- If a value is about 2 or more standard deviations away from the mean in a normal distribution, then it is far from the mean

- The chance that a value that far or farther away from the mean is highly unlikely, given that particular mean and standard deviation

The Standard Normal Distribution

- Also known as the “z” distribution
- Mean is defined to be 0
- Standard Deviation is 1



Values above the mean have positive z-values, values below the mean have negative z-values

The Standard Normal

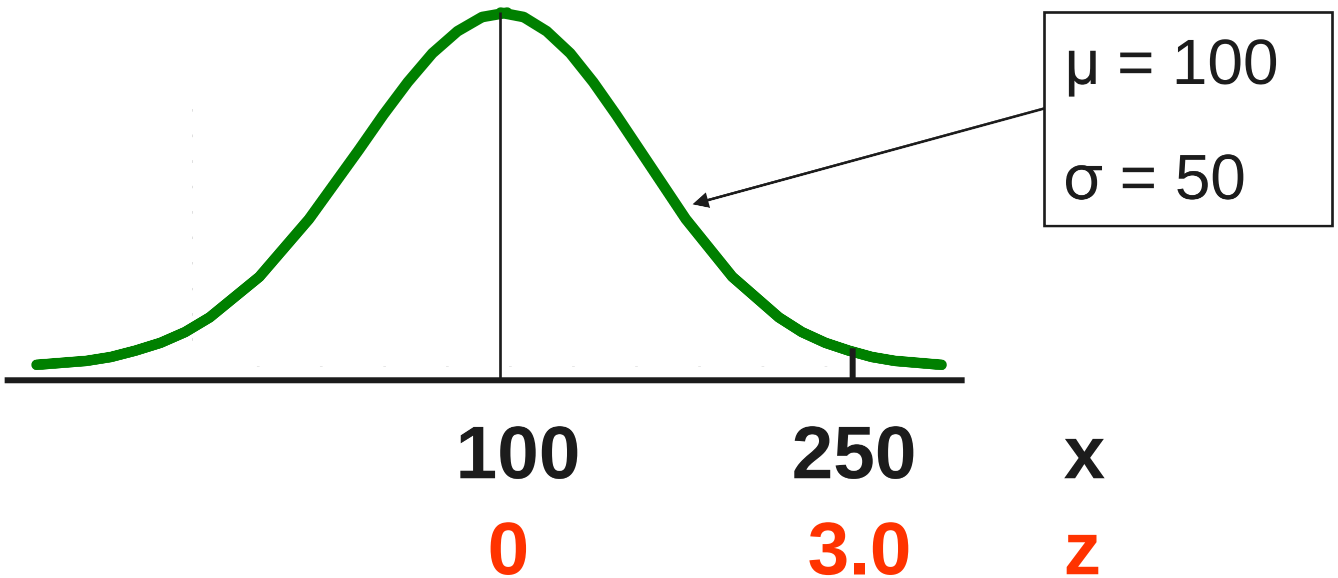
- Any normal distribution (with any mean and standard deviation combination) can be transformed into the standard normal distribution (z)
- Need to transform x units into z units

Translation to the Standard Normal Distribution

- Translate from x to the standard normal (the "z" distribution) by subtracting the mean of x and dividing by its standard deviation:

$$\boxed{z = \frac{x - \mu}{\sigma}}$$

Comparing x and z units

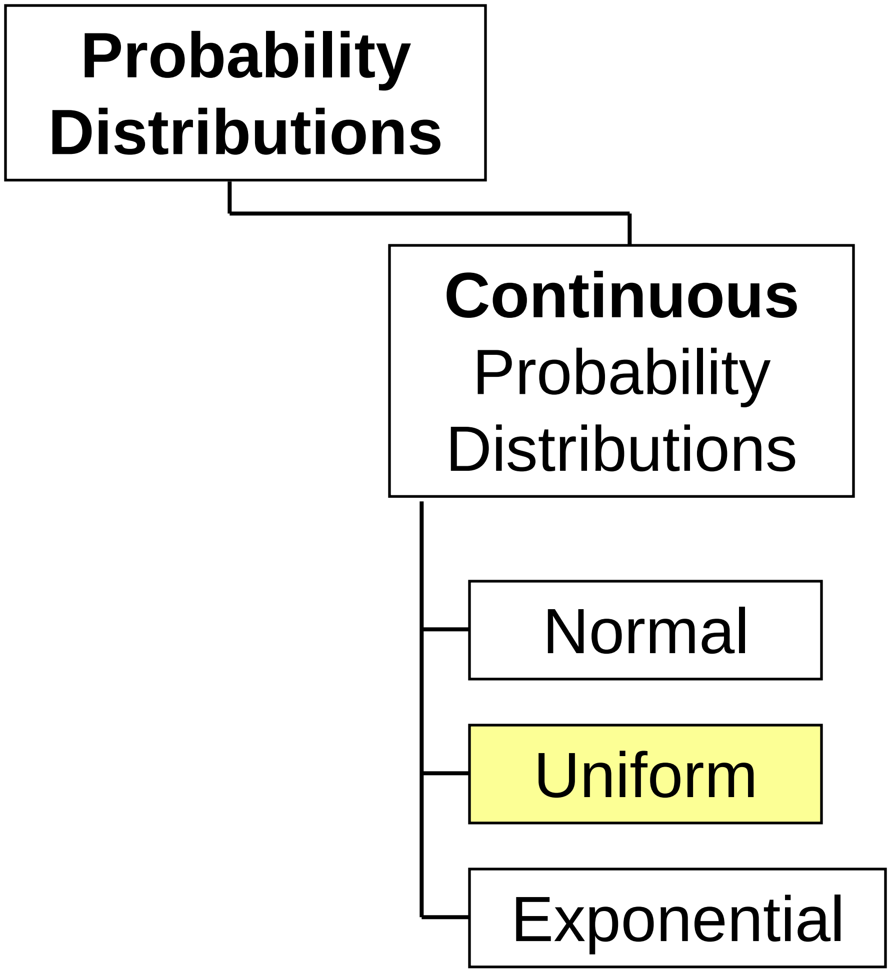


Note that the distribution is the same, only the scale has changed. We can express the problem in original units (x) or in standardized units (z)

Excel Based Examples

Please practice the sample problems that are posted.

The Uniform Distribution



The Uniform Distribution

- The uniform distribution is a probability distribution that has equal probabilities for all possible outcomes of the random variable

The Uniform Distribution

(continued)

The Continuous Uniform Distribution:

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

where

f(x) = value of the density function at any x value

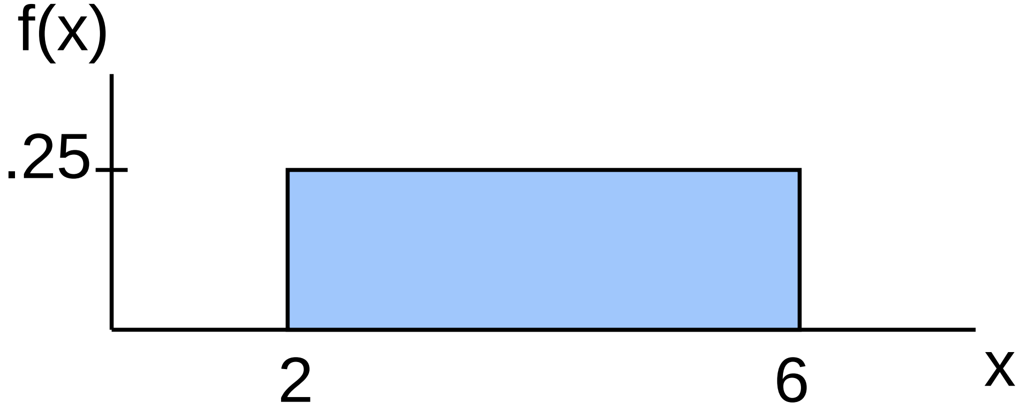
a = lower limit of the interval

b = upper limit of the interval

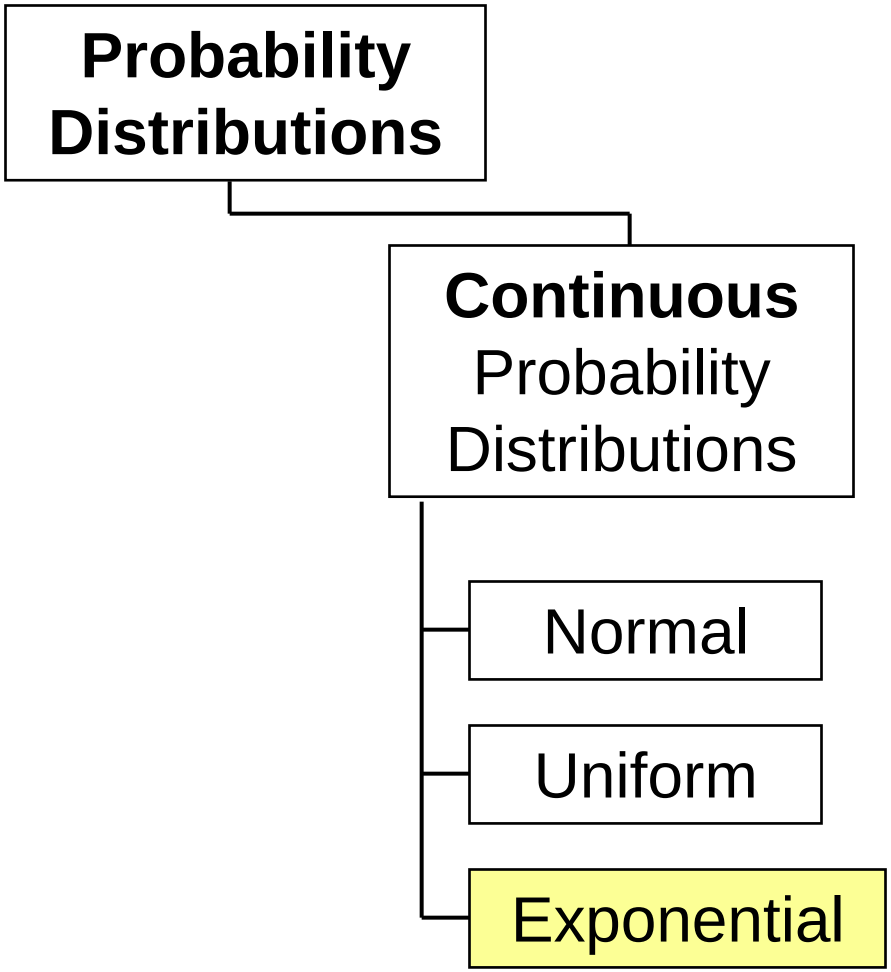
Uniform Distribution

Example: Uniform Probability Distribution Over the range $2 \le x \le 6$:

$$f(x) = \frac{1}{6-2} = .25 \quad \text{for } 2 \le x \le 6$$



The Exponential Distribution



The Exponential Distribution

- Used to measure the time that elapses between two occurrences of an event (the time between arrivals)

  - Examples:
    - Time between trucks arriving at an unloading dock
    - Time between transactions at an ATM Machine
    - Time between phone calls to the main operator

The Exponential Distribution

- The probability that an arrival time is equal to or less than some specified time a is

$$\boxed{P(0 \le x \le a) = 1 - e^{-\lambda a}}$$

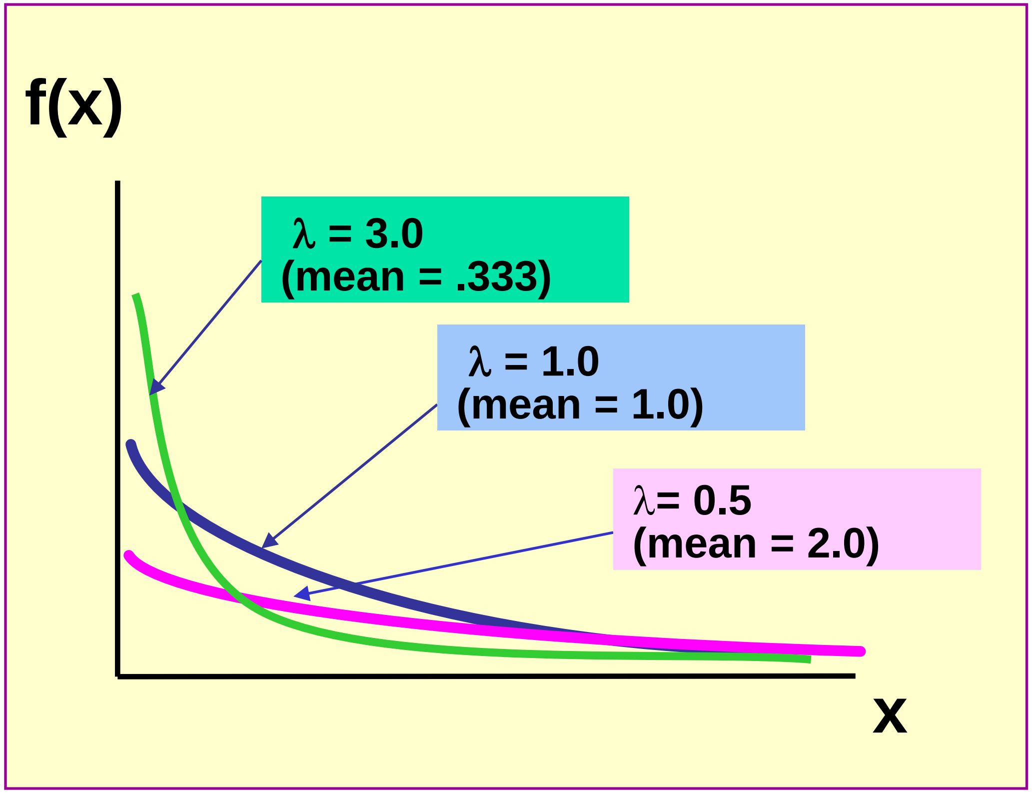
where $1/\lambda$ is the mean time between events

Note that if the number of occurrences per time period is Poisson with mean λ, then the time between occurrences is exponential with mean time $1/\lambda$

Exponential Distribution

(continued)

- Shape of the exponential distribution



Example

Example: Customers arrive at the claims counter at the rate of 15 per hour (Poisson distributed). What is the probability that the arrival time between consecutive customers is less than five minutes?

- Time between arrivals is exponentially distributed with mean time between arrivals of 4 minutes (15 per 60 minutes, on average)
- $1/\lambda = 4.0$, so $\lambda = .25$
- $P(x < 5) = 1 - e^{-\lambda a} = 1 - e^{-(.25)(5)} = \boxed{.7135}$

Module Summary

- Reviewed key continuous distributions
  - normal, uniform, exponential
  - Examples using Excel (sample problems)